

Critical street links for demand responsive feeder transit services

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ABSTRACT

This paper presents a methodology to identify and locate critical links in a grid street network system for feeder transit services. A 'critical' link can be defined as a link that—when eliminated from or appended to an existing network—would cause the largest change in the network connectivity and consequently in the transit performance. The most significant contribution of this study is to present a simple analytical approach to locating the critical link(s) for a grid street network system of any size with uniform passenger demand across the service area. The distance between demands points have been used as the basic measure of impedance in the final derived closed-form equations. Easily computable formulas for identifying the critical links have been validated by simulation analyses with the street network system from the City of St. Joseph, Missouri. Useful insights from the analytical derivations and simulation results indicate a monotonic decrease in link criticality when moving from the centrally located links to those located at the periphery in a grid street network system.

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1. Introduction

Transportation networks play a crucial role in enhancing social and economic growth of a region and facilitating movement of goods and people by connecting two spatially separated regions. In recent years, transportation networks have seen tremendous growth in congestion and a decrease in mobility as well as performance due to the increase in the number of vehicles added to the roadways. The result is an increase in congestion, increase in travel time for goods, gas consumption, and emissions. These thwart the economic growth and business development in a region.

Amongst different street types found in the United States, the 'interstates' are considered to be the most prominent facilitators of economic growth, as these are the most preferred routes for long distance transport within urban and suburban areas. One of the main reasons for their preference is due to the high vehicular speed limits that these interstates offer. However, the average vehicular speed over these interstates decreases as one travels from a rural to an urban city just due to congestion. In fact, over the past few years, these highway systems have seen the greatest amount of delays compared to the arterial streets solely due to congestion (Schrank, Loman, & Eisele, 2011). Often, these interstates reach full capacity in the urban areas, and then traffic spills over onto the adjacent arterials and minor streets, disabling important portions of a transportation network.

Usually, interstates and freeways within a city can be jammed due to an increased number of cars and truck movements taking

place simultaneously during the same time of day, for example, during peak hours. The adjacent arterials also become sensitive to jams during these times, further lowering the level-of-service of the overall transportation network system. Poor network efficiency often prevents emergency services (such as movement of fire trucks, and ambulances) to reach their required destination on time. Several other types of disruptions, such as landslides, snowfall, and storms also lead to network failures, though not as regularly as incidents or lane maintenance activities. Restoring operations for traffic movement over a heavily congested interstate or arterial is important to maintain minimal delays to the flow of people, goods, and emergency services. While it is easy to tag a transportation link with regular congestion on a daily basis as problematic, it is difficult to predict and identify other links where potential failures might make greater impacts on the overall transportation network performance. Thus, a system or tool that can provide planners and engineers with an advance mechanism for identifying the most vulnerable/important links in a network would certainly be quite helpful.

Among different street types, arterials and local roads are often the ones that are most frequently used by people for trips to work, school, shopping, etc. Congestion or blockage on arterials often affects a large portion of the population residing within the vicinity of these streets. Most businesses, malls, services, etc. often situate themselves alongside the arterials which make these streets busier than the others. This demands improved mobility of arterials at all times.

In reality, certain links in a transportation network exhibit improved mobility due to lesser vehicular traffic as compared to other links, while some other links suffer from perpetual congested

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conditions throughout the day. Constant evaluation and monitoring of congested links for their performance becomes more important when they lie on a major transit line, such as a major bus route, as its failure would affect a large number of passengers. Advance knowledge of traffic conditions over these links can be vital in helping transit buses to reroute in case of deteriorating traffic conditions. Although, all types of public transit systems benefit immensely by having an advance rerouting policies in case of link failures, a transit system that generally operates using a flexible routing strategy would certainly benefit the most.

There are different forms of flexible transit systems that operate in the United States and within suburban areas. These flexible services serve as a means for providing the first/last mile transport connectivity, a lack the U.S. Department of Transportation recently recognized is responsible for transit ridership decline in the United States. Some examples of these flexible transit systems that are currently in operation are the Call-n-Ride (CnR) operated by the Denver Regional Transportation District, CO; Demand-Responsive Connector (DRC) operated by Omnitrans, CA; and Demand Responsive Transit (DRT) operated by Mason County Transportation Authority, WA (Potts, Marshall, Crockett, & Washington, 2010). The performance of these flexible transit systems is highly dependent on the street network connectivity and the role each street link plays in facilitating shuttle operations (Chandra & Quadrioglio, 2013a). Thus, a flexible transportation service would surely benefit by having advance knowledge of critical/vulnerable links in a network for rerouting purposes in case of link failures or closures.

In this paper, we focus on identifying the critical link(s) in a grid street network by exploiting the relationship that exists between performance of a DRT and the existing street network topology.

A demand responsive transport is characterized by flexible routing and scheduling of a shuttle operating in shared-ride mode. The shuttle operates by picking up and dropping off passengers from their request locations with a common depot or terminal. In many areas, DRT is also known as Dial-a-Ride Transit (DART) and provides a convenient public transport service in areas of low passenger demand (Stein, 1978). DRT is often operated as a form of socially necessary transport by the local transit authorities or by private operators. However, the main aim of the operators is to ensure best shuttle performance and service standards for their passengers.

2. Literature review

A good street connectivity reduces shuttle travel times, which makes transit attractive to passengers. Quantification of street connectivity by an index that is also sensitive to network degradations would be quite useful in deciding whether a street system is suitable for operating the transit service with desired performance. Jenelius, Petersen, and Mattsson (2006) derived several 'link importance' indices and 'site exposure' indices based on the increase in generalized travel cost when links are closed or disabled. Scott, Novak, Aultman-Hall, and Guo (2006) proposed a new Network Robustness Index to identify critical links and evaluate network performance considering network flows, link capacities, and network topology. Similarly, Knoop, Snelder, and Van Zuylen (2007) developed indicators to determine vulnerable parts of a network by simulating the network flows with an incident on each of the links.

Network analyses for criticality and disruptions have also been extensively studied due to threats involved in the potential failures of large-scale infrastructure systems. A real-life example from the past is the I-35 bridge collapse in Minneapolis, Minnesota, in 2007 that triggered research focused on assessing impacts for other bridges in the United States. In a populated urban city, disruptions

of these kinds are certainly catastrophic. Besides causing loss to life and property, a considerable amount of time is spent in restoring the failed system, which causes unwanted congestion on other adjacent links of the network. A number of research articles can be found that are focused on understanding single link failure cases by considering an increase in travel time or travel distance for commuters as a vital component in the determination of critical links (Sullivan, Novak, Aultman-Hall, & Scott, 2010). In this regard, while Chen, Yang, Lo, and Tang (2002) and Asakura, Hato, and Kashiwadani (2003) have studied specific links for capacity failures, others, such as Sohn (2006) and Berdica and Mattsson (2007), have studied the impact of different kinds of disturbances on links and their consequent effects on the transport systems. In all these studies, links are either considered to be fully functional or closed/absent from the network. Single link failures can be considered as a form of binary technique to measure link criticalities over a network by eliminating the links of a network and analyzing the impact and performance for the whole network. However, these kinds of analyses are quite tedious and computationally difficult if an infinitely large number of links are present, such as those found in an urban city with several links and nodes. There are also other different perspectives to examining critical links in a network as well. The network reliability approach (e.g., Matuszko, Murray, & Grubisic, 2007; Iida, 1999; Clark & Watling, 2005), robustness testing (e.g., Wilson, 2007; Nagurney & Qiang, 2007), and travel time uncertainty analysis (e.g., Bates, Polak, Jones, & Cook, 2001) of transportation networks for link failures are some of them. Several other road network vulnerability studies have been performed based on economic impacts of link failures due to natural disasters (Ham, Kim, & Boyce, 2005; Tsuchiya, Tatano, & Okada, 2007).

Taylor, Sekhan, and D'Este (2006) performed vulnerability analysis on road networks by considering the socioeconomic impacts of network degradation. Several standard indices of accessibility were considered which included a generalized travel cost, the Hansen integral accessibility index, and the ARIA index used in Australia. However, the evaluation is not explicitly based on the impacts on vehicular performances for any given link failure. Moreover, in these types of studies, often a solvable approximation is needed for identifying the most vulnerable arc or link in a network using some algorithmic approach. Ball, Golden, and Vohra (1989) used the most vital arcs problem (MVAP) of finding a subset of arcs such that whose removal from the network results in the greatest increase in the shortest distance between two specified nodes. Corley and Sha (1982) used algorithms to identify the most vital links (or nodes) in a weighted network whose removal from the network resulted in the greatest increase in the shortest distance between two specified nodes.

However, most link failure studies need programming software to code the network to compute changes in travel time by partially or fully closing the links for travel. No doubt this process of locating the most critical link is computationally expensive, even for simple and symmetrically existing real grid street networks that are plentiful in the United States.

This paper presents some closed form results that would help planners and engineers decide quickly on the most critical link for a grid street network of any size and dimension without involving exhaustive computations or simulations. Critical links of a grid based network are identified as those that when removed/closed would cause the largest drop in performance for a general demand responsive transit system. Later, simulation experiments are also performed to validate analytically determined critical link(s) in the network.

Note it is not necessary to remove or disable links in a network. A grid street system that has no links at all, such as found in cul-de-sacs, would exhibit the same characteristics with links being removed in the network. This paper also serves to analyze grid street systems that have missing links in reality, though the focus of this

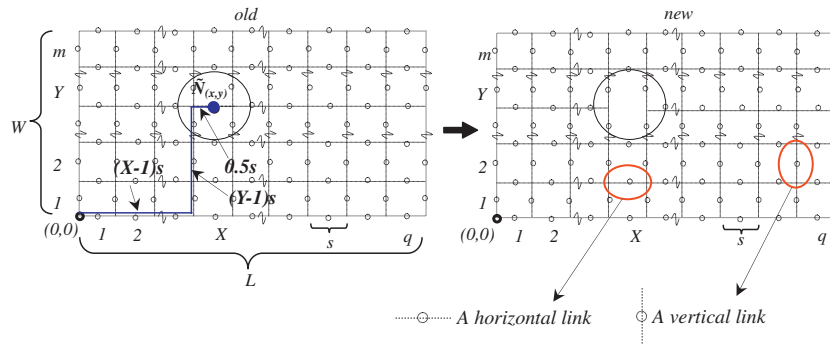


Fig. 1. Removal of a link to create a 'new' network from an 'old' network.

paper is to analyze performance effects of DRT due to complete link removals.

3. Methodology

3.1. Street Network Set-up

A general grid street network system is selected that has a service area of length, L and width, W ; with each block side length equal to s . A grid street network system is ideally selected as several other forms of street networks such as the rectangular and cul-de-sacs, can be constructed by eliminating or adding links to this uniform grid street network system. Each of the links is identified using distinct sets of on-demand nodes, such as $\tilde{N}_{(x,y)}$ for a horizontal link in Fig. 1 (with variables x and y explained later). The on-demand node $\tilde{N}_{(x,y)}$ marks the horizontal link as a solid circle in the 'old' network while other links are marked with nodes using empty circles. We can have similar on-demand node notation for a vertical link as well. Note that there is an inherent symmetry involved in these notations whether the node is located on a horizontal or a vertical link – as every horizontal link in the grid network shown in Fig. 1 becomes a vertical link for 90° rotation along the plane of the paper. Thus, we focus on analyzing an on-demand node on a horizontal link and any analysis performed on a node $\tilde{N}_{(x,y)}$ located on a horizontal link can be extended to get results/equations for a node placed on a vertical link due to just a swapping in terminologies and variables in notations resulting from the 90° rotation.

Now consider the grid street network system with horizontal and vertical links as shown in Fig. 1. The 'new' network in Fig. 1 is formed by eliminating one of the horizontal links (which has $\tilde{N}_{(x,y)}$ from the 'old' network. The on-demand node on the horizontal link designated as $\tilde{N}_{(x,y)}$, is also deleted by default. As per Fig. 1, the node $\tilde{N}_{(x,y)}$ is such that $x = (X-1)s + 0.5s = (X-0.5)s$ and $y = (Y-1)s$, where $X \in \{1, 2, \dots, q\}$ with $q \approx (L/s)$ and $Y \in \{1, 2, \dots, m, m+1\}$ with $m \approx (W/s)$. Using all the range of values of x and y would effectively mark all on-demand nodes on each of the horizontal links in the network. Also, with variable x and y , it simply means that the on-demand node $\tilde{N}_{(x,y)}$ is located at a horizontal shortest path distance of x units (equal to $(X-0.5)s$) plus a vertical shortest path distance of y units (equal to $(Y-1)s$) from the origin $(0,0)$.

Essentially, the on-demand nodes (whether on horizontal or vertical links) are such that they represent the pick-up/drop-off points for the passengers in a random demand distribution setting. Mid-block locations are selected for possible shuttle stops, as unnecessary interference with the upstream traffic at the intersections is avoided (Fitzpatrick, Hall, Perkinson, Nowlin, & Koppa, 1996). These mid-block stops are called on-demand stops in this study, as the shuttle would only visit these nodes if a service request is made at these nodes. On-demand stop designations at

street mid-blocks can also be understood to be average locations of random demands occurring on a street.

The block size's of the grid street is such that it is within the desired walking capacity of the passengers to a transit stop. Suitable values of block sizes that could be within a pedestrian walking capacity can be decided based on research results by O'Sullivan and Morrall (1996).

3.2. Preliminary formulas

It has been shown by Chandra and Quadrioglio (2013a) that the DRT performance is inversely proportional to the total demand weighted distance (T) among all on-demand nodes (total N). The expression for T for a variable demand rate λ_i at each on-demand node i is,

$$T = \sum_i \left(\lambda_i \sum_j d_{ij} \right), \quad \forall i, j \in \{1, 2, \dots, N\}, \quad j \neq i \quad (1)$$

where d_{ij} is the shortest path between nodes i and j to be actually driven by the shuttle. We can also substitute the distance variable in (1) with,

$$d_{ij} = (V_{ij} t_{ij}) \quad (2)$$

where V_{ij} is the average speed of the shuttle and t_{ij} is the travel time of the shuttle between nodes i and j . For each link between nodes i and j , the distance d_{ij} is actually known as being the part of the infrastructure and with speed V_{ij} that can also be approximated by the posted speed limit over the link. Thus, travel time over each link can be estimated and it would be more reasonable to work with travel times as this is a primary measure of shuttle performance. There are alternate ways of estimating travel times over the links using traffic assignment which might require data related to link characteristics such as flow, capacity, number of lanes, and intersections' type. Further, calibrations are needed for parameters if a BPR function is used (Easa (1991)). Added to these, flow and other traffic variables are quite dynamic in nature which poses challenge for easy travel time data estimation. However, as we are focusing on DRT services which are mostly operated within a low-demand residential areas (Potts et al., 2010), travel times are directly related to the distance traveled with average speed often equal to the posted speed limit. Thus, the assumption of working with 'distance' for DRT would not deviate significantly for the results if 'travel time' were used. In agreement with this, the operators and stakeholders can even do away with the challenges involved in obtaining travel time data to determine critical links on the network. Thus, continuing to work with distance as the key variable, the demand weighted average distance between two on-demand nodes is expressed as,

$$T_{avg.} = \frac{1}{A} \sum_{i=1}^N \left(\lambda_i \frac{\sum_{j=1}^N \lambda_j d_{ij}}{A - \lambda_i} \right) \quad (3)$$

where $A = \sum_{i=1}^N \lambda_i$, i.e. summation of all demand rates. For a given grid street network, the change in T is computed using a variable number of link removals from the network. Each link removal also causes an elimination of an on-demand node (along with its demand rate) from the network since the nodes are located on each of the links. Thus, for ‘ a ’ number of link removals, the change in demand weighted average distance (Z) among remaining on-demand nodes is given by,

$$Z = \left(\frac{1}{A - \sum_{k=1}^a \lambda_k} \sum_{i=1}^{N-a} \left(\lambda_i \frac{\sum_{j=1}^{N-a} \lambda_j d_{ij}}{A - \sum_{k=1}^a \lambda_k - \lambda_i} \right) \right) - \left(\frac{1}{A} \sum_{i=1}^N \left(\lambda_i \frac{\sum_{j=1}^N \lambda_j d_{ij}}{A - \lambda_i} \right) \right) \quad (4)$$

Note that the change in demand weighted average distance in (4) is very generic and applicable for any network type and not necessarily for a grid network system as introduced earlier. However, we arrive at a very complex equation shown in (4) that becomes mathematically intractable for multiple nodes. Thus, our aim to provide transit planners with a handy (even though approximated) tool readily usable by transit operators to make decisions for shuttle rerouting in case of possible street link failures would be challenged.

Nevertheless, an analysis of the street systems for some 24 different DRT services surveyed by Koffman (2004) and Potts et al. (2010) shows that several residential areas served by these kinds of shuttles are evenly laid out as a grid, a very common pattern in the United States, where we reasonably believe that assuming spatially uniform demand distribution is quite acceptable and would lead our model to provide good results. The streets of the City of St. Joseph in Missouri, USA, are a good example of a grid-like street pattern (see Fig. 2). St. Joseph Transit provides a flexible transport for the passengers throughout the city.

Although we fully realize the drawback involved with this assumption (becoming too strong for uneven geometries and/or not-uniform distributions), we also wanted to preserve the simplicity of the theoretical formulations to provide usable and handy results. Similar spatially uniform demand distribution has been assumed in a variety of literatures with regard to DRT studies (such as Quadrifoglio, Hall, & Dessouky (2006), Zhao & Dessouky (2008) among others) to arrive at key results very useful for DRT operations.

Thus, with an assumption of uniform passenger distribution over the service area having equal demand rates for all nodes and in specific working with just the grid networks, we have (4) simplified as,

$$Z = \frac{T_f}{(N-a)(N-a-1)} - \frac{T_i}{N(N-1)} \quad (5)$$

where T_f and T_i are the final and initial T after the link removals from the network.

Note that, while uneven spatial demand distributions would challenge our model, time-wise variable demand, with higher/lower levels throughout the day, intuitively would not affect the identification of critical links. This is verified by our validation in the next sections.

Rewriting (1), we have,

$$Z = \frac{1}{N(N-1)} \left(\frac{T_f}{\left(1 - \frac{a}{N}\right) \left(1 - \frac{a}{N-1}\right)} - T_i \right) \quad (6)$$

For $N \gg a + 1$, the right hand side terms for Z in (6) can be approximated as,

$$Z \approx \frac{1}{N(N-1)} (T_f - T_i) \quad (7)$$

The expression in (7) shows that average change in distance between two on-demand nodes in a grid network structure of Fig. 1 is directly proportional to change in total distance among all nodes, provided the number of nodes removed are much less compared to total number of on-demand nodes present in the network.

There are three different ways to remove links in a grid street network set-up of Fig. 1 – (i) only links placed horizontally are removed, (ii) only links placed vertically are removed, or (iii) a combination of both horizontal and vertical placed links are removed. Now, let the node labeled as $\tilde{N}_{(x,y)}$ is removed from the network. The change in T consisting of all street based shortest path distances from node ‘ $\tilde{N}_{(x,y)}$ ’ is the change in distance between two on-demand nodes and consists of two components: decrease in T (say $S_{(x,y)}$) and increase in T (say, $R_{(x,y)}$). The expressions for $S_{(x,y)}$ and $R_{(x,y)}$ for a single link removal case are derived in the next sections.

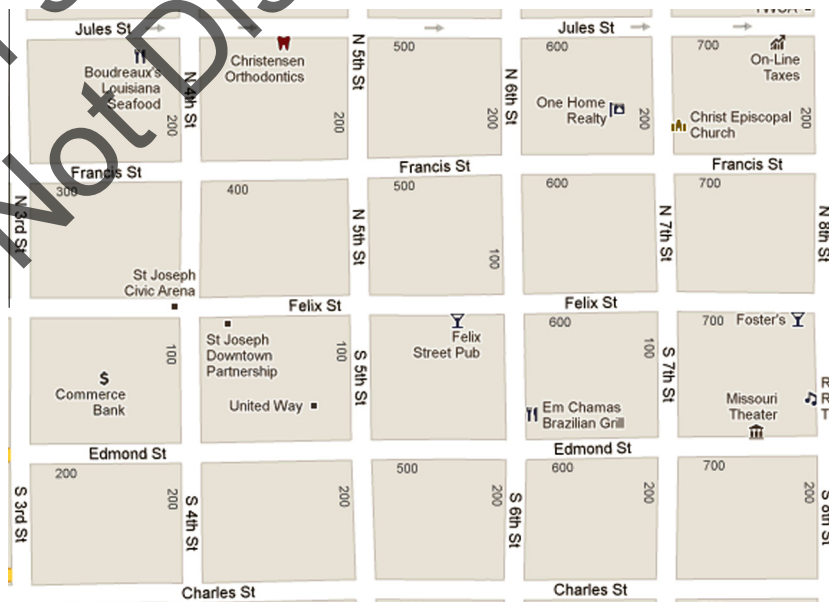


Fig. 2. Grid street patterns around the City of St. Joseph, MO. Source: Google Maps.

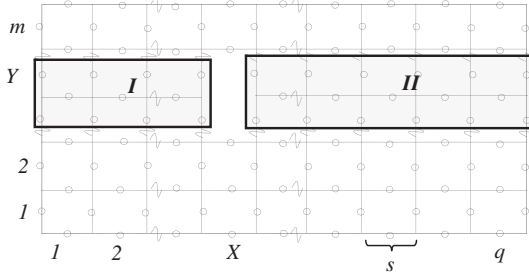


Fig. 3. Enclosed nodes for computing $R_{(x,y)}$.

3.3. Link removal-increase in average travel distance

Elimination of a link results in an increase in total distance for some of the nodes in the network. The sketch in Fig. 3 is used to derive the expression for the increase in distance, $R_{(x,y)}$.

Each of the bottom nodes and uppermost aligned horizontally in region I is at an increased distance of '2s' with the nodes in region II after the link removal. Thus, there is an overall increase of total distance of $2X\{(2s)(q - X) + s\}$. For the middle layer of nodes, we have a total increase of s, 2s and s with the top, middle, and bottom horizontal aligned nodes of region B, respectively. Thus, there is an overall increase of total distance of $(X-1)\{(s + s)(q-X + 1) + 2s(q-X)\}$ for the middle horizontal aligned nodes of I. Similarly, we find the total an increase in distance of nodes of region II. Using the above discussion, $R_{(x,y)}$ (i.e., increase in T due to link and on-demand node removal) can be written as,

$$R_{(x,y)} = 2X(2(q - X) + 1)s + 2(X - 1)((q - X) + 1)s + 2(X - 1)(q - X)s + 2(q - X + 1)(2(X - 1) + 1)s + 2X(q - X)s + 2(X - 1)(q - X)s = 4(4qX - 4X^2 + 4X - 2q - 1)s \tag{8}$$

Similarly, let $G_{(v,w)}$ be the increase in T due to a vertically aligned link's removal from the network. The resulting expression is not included here as it is easily obtained from (8) by replacing q with m, Y with X, and W with Y, respectively.

3.4. Link removal-decrease in average travel distance

We derive a general expression for grid-based distances from a given on-demand node located anywhere in the network to all other nodes in the network. For this, let us define a 'target' node to which we compute the sum of total distances from all on-demand nodes. This target node (shown as a solid circle; empty circles are on-demand nodes or just nodes) is located at the lower horizontal line of the bottom corner grid of the street network system, as Fig. 4 shows.

In the sketch of Fig. 4, it can be said that there are a total of $(2m + 1)$ series of nodes that are distributed horizontally, if

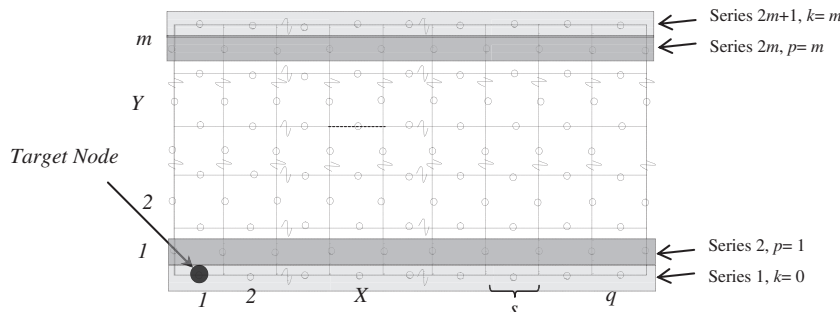


Fig. 4. Sketch depicting the 'target' node in the street network system.

counted on top of each other in a vertical direction (see Fig. 4 for illustration) Let $T_{(2m+1-2k)}$ denote the sum of distances from the target node to a series nodes belonging to 'k' ($k \in \{0, 1, 2, \dots, m\}$). The last sum being $T_{(2m+1)}$ for $k = 0$. Also, let $T_{(2m+2-2p)}$ stand for the sum of distances from the target node to series of nodes belonging to 'p' ($p \in \{1, 2, \dots, m\}$). The last sum is $T_{(2m)}$ for $p = 1$. Also, notice that series with k or p appear alternately as evident from Fig. 4. Thus, using simple arithmetic progressions for summing distances between target node and to each specified series, in terms of q and m, we have,

$$T_1 = \left\langle \frac{q(q - 1)s}{2} + s + qms \right\rangle \text{ at } k = m \tag{9}$$

$$T_2 = \left\langle \frac{q^2s}{2} + \frac{s}{2} + (q + 1)\left(ms - \frac{s}{2}\right) \right\rangle \text{ at } p = m \tag{10}$$

$$T_{2m} = \left\langle \left(\frac{q^2s}{2} + \frac{s}{2}\right) + (q + 1)\left(ms - \frac{(2m - 1)s}{2}\right) \right\rangle \text{ at } p = 1 \tag{11}$$

$$T_{2m+1} = \left\langle \left(\frac{q(q - 1)s}{2} + s\right) + q\left(ms - \frac{2ms}{2}\right) \right\rangle \text{ at } k = 0 \tag{12}$$

Thus, summation of all the above terms in T_e with $e = \{1, 2, \dots, 2m + 1\}$ would give the sum of all distances from all the nodes of the street network system to the target node. It can be easily computed using some rearrangement of terms as an arithmetic progression series and we can write,

$$\sum_{k=0}^m T_{2m+1-2k} + \sum_{p=1}^m T_{2m+2-2p} = \sum_{i=1}^{2m+1} T_i = \left\langle qm^2s + (m^2s/2) + mq^2s + \frac{3ms}{2} + \frac{q(q - 1)s}{2} \right\rangle \tag{13}$$

Using the above result for sum of the distances from all nodes of the network to the target node, we can derive an expression for any general on-demand node to all other on-demand nodes. First, we notice that we can treat any intermediate node in the network system as a 'shifted target node' with the total area divided into four regions such as A, B, C, and D, as shown in Fig. 5.

The shifted target node lies such that there is an overlapping area of A, B, C, and D that needs to be accounted for (see Fig. 5). Thus, the sum of distances from all the nodes of each of the respective four regions (say $S_{(x,y)}$) to this target node has been derived and written using (13) in final form as,

$$S_{(x,y)} = \left(\begin{aligned} &(q + 2)(m - Y + 1)^2s + m(q - X + 1)^2s + (3m + 2)s + \frac{(q - X)(q - X + 1)s}{2} \\ &+ (q + 2)(Y - 1)^2s + mX^2s + \frac{(X - 1)Xs}{2} - \left(\frac{Y(Y - 1)s}{2} + \frac{(3m - 3Y + 5)(m - Y + 2)s}{2} \right) \end{aligned} \right) \tag{14}$$

The corresponding expression for decrease in T can also be obtained for the removal of a vertical aligned link (similar to increase in T as $G_{(v,w)}$) in the 'old' network of Fig. 1. Let this decrease be denoted by $H_{(v,w)}$, replacing q with m (and vice-versa), V with X, and W with Y, respectively.

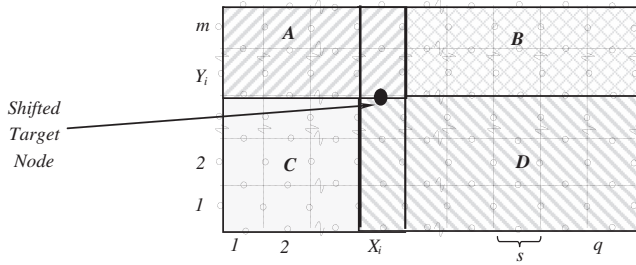


Fig. 5. Split regions to create target node for any intermediate node.

3.5. Determination of critical links

In reality, network inefficiency is attributed to its failure resulting from a group of links that are congested or jammed. Studying the criticality of a single link one at a time in isolation would certainly be insufficient. This section describes the methodology for locating a group of links that are together critical in the grid network. Let a_1 and a_2 be the numbers of horizontally and vertically aligned links collectively removed or disabled from the old network of Fig. 1, respectively. A generalized form of the expression for net increase in T , (Z_d), can be written as,

$$Z_d = \sum_{i=1}^{a_1} R_{(X_i, Y_i)} + \sum_{j=1}^{a_2} G_{(V_j, W_j)} - 2 \left(\sum_{i=1}^{a_1} S_{(X_i, Y_i)} + \sum_{j=1}^{a_2} H_{(V_j, W_j)} - \sum_{e=1}^{a_1+a_2} \sum_{f=e}^{a_1+a_2} D_{e,f} \right) \quad (15)$$

where $X_i = X \in \{1, 2, \dots, q = (L/s)\}$, $V_j = V \in \{1, 2, \dots, m = (W/s)\}$, $Y_i = Y \in \{1, 2, \dots, m = (W/s)\}$, and $W_j = W \in \{1, 2, \dots, q = (L/s)\}$. The term $D_{e,f}$ = grid street-based distance between on-demand nodes e and f and it is also a function of the variables X_i, Y_i, V_j and W_j . The multiplication term '2' in Eq. (15) is the back and forth distance which is to be counted twice between the removed node and any other node.

Assuming all the variables X_i, Y_i, V_j and W_j as continuous, the gradient of Z_d is

$$\Delta Z_d = \left[\frac{\partial Z_d}{\partial X_1}, \dots, \frac{\partial Z_d}{\partial X_{a_1}}, \frac{\partial Z_d}{\partial Y_1}, \dots, \frac{\partial Z_d}{\partial Y_{a_1}}, \frac{\partial Z_d}{\partial V_1}, \dots, \frac{\partial Z_d}{\partial V_{a_2}}, \frac{\partial Z_d}{\partial W_1}, \dots, \frac{\partial Z_d}{\partial W_{a_2}} \right] \quad (16)$$

We only show the expression for the two terms of the gradient; i.e., $\frac{\partial Z_d}{\partial X_i}$ and $\frac{\partial Z_d}{\partial Y_i}$, while the expressions for other terms can be derived by replacing X with V, Y with W , and m with q (and vice-versa) appropriately. Writing the partial derivative of Z_d with respect to X_i , we get,

$$\frac{\partial Z_d}{\partial X_i} = (18q + 4mq + 4m + 18 - 36X_i - 8mX_i)s + 2 \frac{\partial}{\partial X_i} \left(\sum_{e=1}^{a_1+a_2} \sum_{f=e}^{a_1+a_2} D_{e,f} \right) \quad (17)$$

The critical point for (17) is,

$$X_i = \frac{(q+1)}{2} + \frac{1}{(18+4m)} \frac{\partial}{\partial X_i} \left(\sum_{e=1}^{a_1+a_2} \sum_{f=e}^{a_1+a_2} D_{e,f} \right) \quad (18)$$

and,

$$\frac{\partial^2 Z_d}{\partial X_i^2} = (-36 - 8m)s + 2 \frac{\partial^2}{\partial X_i^2} \left(\sum_{e=1}^{a_1+a_2} \sum_{f=e}^{a_1+a_2} D_{e,f} \right) \quad \forall i \in \{1, 2, \dots, a_1\} \quad (19)$$

Also,

$$\frac{\partial Z_d}{\partial Y_i} = (4qm - 8qY_i + 8q + 2m - 4Y_i + 4)s + 2 \frac{\partial}{\partial Y_i} \left(\sum_{e=1}^{a_1+a_2} \sum_{f=e}^{a_1+a_2} D_{e,f} \right) \quad (20)$$

The critical point for (20) is,

$$Y_i = \frac{(m+2)}{2} + \frac{1}{(2+4q)} \frac{\partial}{\partial Y_i} \left(\sum_{e=1}^{a_1+a_2} \sum_{f=e}^{a_1+a_2} D_{e,f} \right) \quad (21)$$

and,

$$\frac{\partial^2 Z_d}{\partial Y_i^2} = (-8q - 4)s + 2 \frac{\partial^2}{\partial Y_i^2} \left(\sum_{e=1}^{a_1+a_2} \sum_{f=e}^{a_1+a_2} D_{e,f} \right) \quad (22)$$

$$\forall i \in \{1, 2, \dots, a_1\}$$

Analyzing Eqs. (18) and (21), we can deduce that the maximum change in T would occur if the centrally located links are closed shown by $X_i \approx \frac{(q+1)}{2}$, and $Y_i \approx \frac{(m+2)}{2}$, since these terms should be positive integers. This can also be easily justified with assumption that $\frac{(q+1)}{2} \gg \frac{1}{(18+4m)} \frac{\partial}{\partial X_i} \left(\sum_{e=1}^{a_1+a_2} \sum_{f=e}^{a_1+a_2} D_{e,f} \right)$ and $\frac{(m+2)}{2} \gg \frac{1}{(2+4q)} \frac{\partial}{\partial Y_i} \left(\sum_{e=1}^{a_1+a_2} \sum_{f=e}^{a_1+a_2} D_{e,f} \right)$, especially when m and q are large. The Hessian matrix for Z_d can also be shown to be negative definite with some example terms $\frac{\partial^2 Z_d}{\partial X_i^2} \approx (-36 - 8m)s < 0$ and $\frac{\partial^2 Z_d}{\partial Y_i^2} \approx (-8q - 4)s < 0$ for $(36 + 8m)s > 2 \frac{\partial^2}{\partial X_i^2} \left(\sum_{e=1}^{a_1+a_2} \sum_{f=e}^{a_1+a_2} D_{e,f} \right)$ and $(8q - 4)s > 2 \frac{\partial^2}{\partial Y_i^2} \left(\sum_{e=1}^{a_1+a_2} \sum_{f=e}^{a_1+a_2} D_{e,f} \right)$.

3.6. Example – single link removal case

We present an example for a 'single' horizontal aligned link removal case in the grid network (example, old network in Fig. 1) with $D_{e,f} = 0$ for given nodes e and f . For a given constant 'Y', the derivative of Z_d with respect to ' $X_i = X$ ' gives, $\frac{\partial Z_d}{\partial X} = 0$ when $X = \left(\frac{q+1}{2}\right)$ and $\frac{\partial^2 Z_d}{\partial X^2} = -8m - 36 < 0$. Therefore, for a given row $Y_i = Y$, Z_d attains a maximum at $X = \left(\frac{q+1}{2}\right)$ for an odd q . For an even number q , we carry out the analysis in a slightly different manner. For a continuous discrete variable 'X,' the expression for Z_d is a discrete convex function and has a maximum at $X = \left(\frac{q+1}{2}\right)$. Thus, Z_d would attain a maximum for an even q at $X = \left(\frac{q}{2}\right)$ or $X = \left(\frac{q}{2} + 1\right)$ or both.

Denoting the terms in $Y_i = Y$ and other constant terms as $f(Y)$ in expression (15), we can do a quick check for one of the two values of X that would give a maximum Z_d . We observe that both for $X = \left(\frac{q}{2}\right)$ and $X = \left(\frac{q}{2} + 1\right)$ we obtain equal expressions of Z_d as,

$$Z_d = \left(4(q^2 - 1)s - 2m\left(\frac{q}{2}\right)^2 - \left(\frac{q}{2}\right)^2 - \left(2m\left(\frac{q}{2}\right)^2 + 2mq + 2m\right)s - \left(\frac{q}{2}\right)^2 \right) + f(Y) = \left(\frac{7q^2}{2} - mq^2 - 2mq - 2m - 4 \right) s + f(Y) \quad (23)$$

Further, if we fix 'X' as constant and find the derivative of Z_d with respect to Y in (15), Z_d attains a maximum at $Y = \left(\frac{m}{2} + 1\right)$ and $Y = \left(\frac{m}{2}\right)$ for even number 'm' and at $Y = \left(\frac{m+1}{2}\right)$ for odd 'm.' This result follows from the symmetry of the street network structure along with the positions of the on-demand nodes.

For a given row in the network system of Fig. 1 (i.e., given Y), Z_d attains a maximum for $X = \left(\frac{q+1}{2}\right)$ and for an odd q . For an even number q , Z_d would attain a maximum both for $X = \left(\frac{q}{2}\right)$ and $X = \left(\frac{q}{2} + 1\right)$. Further, if we fix 'X' as constant, Z_d attains a maximum at $Y = \left(\frac{m}{2} + 1\right)$ and $Y = \left(\frac{m}{2}\right)$ for even 'm' and at $Y = \left(\frac{m+1}{2}\right)$ for odd 'm.' This result also follows from the symmetry of the street network structure. As an illustration, the solid bars (not to scale) in Fig. 6 represent an approximate magnitude of the variation of Z_d from $Z_{d,max}$ to $Z_{d,min}$ for individual link closure along the cross-section of the network for an odd 'q' and odd 'm.'

In situations when two or more combinations of links are simultaneously eliminated from the 'old' network of Fig. 1, the

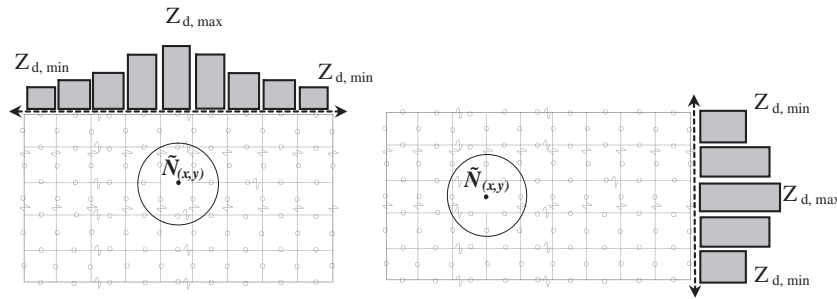


Fig. 6. Variation of Z_d along the cross-section of the street network.

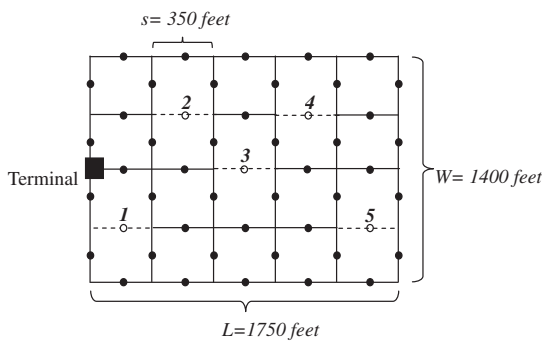


Fig. 7. Sequential link closures/removals 1–2–3–4–5 to create five different sets of street networks.

resulting reduction in vehicular performance would be maximum when most of the centrally located links are removed.

4. Simulation results and discussions

We present some simulation results that validate the determination of critical links as shown above for a single link removal case. A finite size grid street network with identical blocks (as the old network of Fig. 1) is selected with some random links being removed in a sequence each time 1–2–3–4–5 (see Fig. 7). The network in Fig. 7 presents a case where q (or, m) is odd. Hence, five different networks (in each a link missing) are used in the simulation.

A hypothetical DRT service that runs for a variable headway of 7.5–21.5 min (assumed), serving a daily passenger demand of 350

(assumed) within a rectangular area of $L = 1750$ feet, $W = 1400$ feet and $s = 350$ feet (block size for St. Joseph City street network), is put in operation in all five different grid networks (see Fig. 7). The DRT operational times are fixed from 6:30 am to 9 pm and the passengers make random service requests generated from a real travel demand distribution of the US commuters. The distribution is derived from the 2009 National Household Travel Survey (Santos, McGuckin, Nakamoto, Gray, & Lees, 2011), as shown in Fig. 8. The chart shows different trips by purpose for the entire United States. Corresponding cumulative distribution functions (CDF) are developed for the commuter trips, as shown using the ‘Actual Density’ in Fig. 9. The actual CDF is a polynomial of higher degree that is difficult to invert for generating trip travel times. Hence, easily invertible piecewise linear function named as ‘Assumed Density’ are used for random generation of travel times for passengers. In simulation, this is achieved by uniformly generating random numbers between 0 and 1, and estimating travel times using the linear equations under the ‘Assumed Density’ of Fig. 9.

The requests for service usage are accepted between 6 am through 8:30 pm by phone or internet, and each passenger is randomly assigned as pick-up or drop-off request. The spatially random service requests (within a given headway) are distributed uniformly over the on-demand nodes. The order of pick-up/drop-off of passengers, for each of the headways, is carried out using the insertion heuristic (Quadrifoglio, Dessouky, & Palmer, 2007). The street based shortest path distance between two nodes is computed using Dijkstra’s algorithm coded in MATLAB R2008b (MATLAB Documentation, 2011).

The DRT performance can generally be identified as a combination of operating costs and service quality. The service quality is expressed as passengers’ disutility: a weighted sum of expected waiting time and expected in-vehicle travel time (in the ratio of

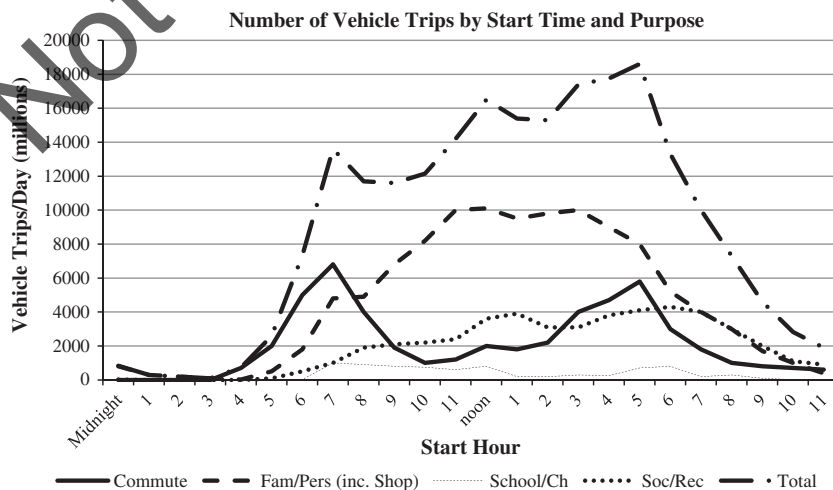


Fig. 8. Trip data for entire US. Source: Santos et al., 2011

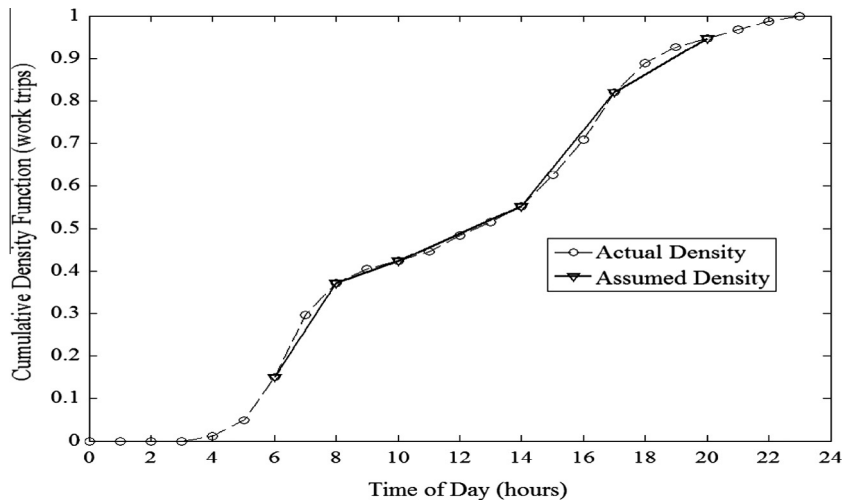


Fig. 9. Cumulative density functions for US work-trips.

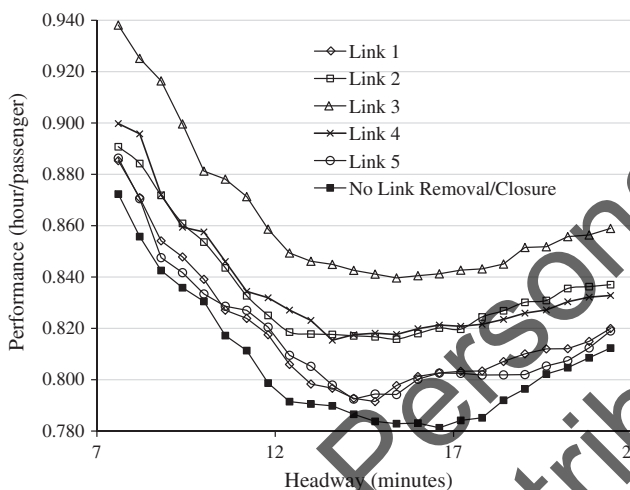


Fig. 10. Performance effects on DRT for the sequential link closures/removals.

1:1.8) typically used for any DRT service (Chandra & Quadrioglio, 2013b; Wardman, 2004). The performance values are obtained from an average of 20 replications for each of the five networks for different links removed, and is shown below in Fig. 10.

The chart in Fig. 10 shows that the removal of Link 3 results in the greatest increase in the average waiting and riding times of the passengers. Link 1 and Link 2 case removal conditions show a very close resemblance in performance with Link 4 and Link 5 case removals, respectively; while for a 'no link removal/closure' condition the performance of the transit shuttle is the best. Thus, simulation results validate our analytical modeling of the critical link(s) identification when q (or m) is odd. We also observed similar simulation outputs that validated the closed-form equations for the location of critical links when q (or m) is even, though not included here.

5. Conclusions

This study presents an analytical model to assess link criticality important for a desirable DRT shuttle performance. The contribution of this work is the closed form equation that identifies the critical links for any grid network size and thus, exhaustive

computations or approximations are avoided. It is found that for a grid street network system, the links located at the interior of the network are more critical than those located at the periphery, and this has been validated through sets of simulation experiments. Though this work mainly focuses on studying a grid network system, a large form of other network structures such as cul-de-sacs and rectangular, can also be analyzed with a similar logic presented in this paper. Comparisons can be made between two different grid street forms that have missing links in reality to assess which network would be more suitable for operating DRT services.

At present, there is scarce research that appropriately addresses the relationship between transit performance and street network topology through a satisfactory street connectivity measure. Thus, the closed form formulas outlined in this paper can be a great aid to transit planners and operators to maximize ridership by efficiently routing shuttles in case of failures, since each link in the network is known for the importance it holds.

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