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ORIGINAL PAPER

2-Vehicle zone optimal design for feeder transit services

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Abstract Feeder transit services perform the crucial first/last mile access to transit by connecting people within a residential area to a major transit network. In this paper, we address the optimal zone design problem faced by planners for feeder transit services with high demands and long length of service area, where a two-vehicle operation is assumed to be adopted in each zone. By balancing customer service quality and operating cost, we develop an analytical model of the system by assuming continuous approximations. Closed-form expressions and numerical procedures are employed to derive the optimal number of zones to aid decision makers in determining the best design as a function of the main parameters. Analytical expressions and results are then validated by simulation analysis.

Abbreviations

The following notations represent model parameters:

λ	average demand in the whole residential area (customer/hour)
α	fraction of customers traveling from the residential area to the city; $1-\alpha$ is the fraction of customers traveling from the city to the residential area
L	length of the residential service area (mile)
W	width of the residential service area (mile)
d	distance between FRT bus stations within a zone (mile)

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a_k	customer cost of walking between a FRT bus station and a house within a zone ($/customer/hour)
a_w	customer cost of waiting at terminals or bus stations ($/customer/hour)
a_w^h	customer cost of waiting at houses ($/customer/hour)
a_v	customer cost of traveling in an on-demand vehicle ($/customer/hour)
a_b	customer cost of traveling in a fixed route bus in the zones ($/customer/hour)
a_B	customer cost of traveling in a major transit vehicle between the city and terminals ($/customer/hour)
F_v	total cost of an on-demand vehicle ($/vehicle/hour)
F_b	total cost of a fixed route bus ($/bus/hour)
v_{wk}	average speed of customer walking (mile/hour)
v_b	average speed of an on-demand vehicle or a fixed route bus (mile/hour)
v_B	average speed of a major transit vehicle (mile/hour)
s	dwelling time of a fixed route bus or an on-demand vehicle (hour)
S	dwelling time of a major transit vehicle at terminals (hour)

The computed variables in the model, that are a function of n and N, are:

$E(T_{wk})$	expected walking time in a zone for pick-up or drop-off customers
$E(T_{wt}^p)$	expected waiting time for pick-up customers in a zone
$E(T_{rd}^p)$	expected ride time for pick-up customers in a zone
$E(T_{rd-B}^p)$	expected ride time for pick-up customers in a major transit vehicle
$E(T_{wt}^d)$	expected waiting time for drop-off customers at a terminal
$E(T_{rd}^d)$	expected ride time for drop-off customers in a zone
$E(T_{rd-B}^d)$	expected ride time for drop-off customers in a major transit vehicle.

1 Introduction

The US Federal Transit Administration often identifies the issue of providing better "first" and "last" mile access to transit as crucial for revitalizing public transportation systems and improving their performance. Urban sprawl is in fact transforming residential areas, which are progressively increasing in size and causing public transit to lose its effectiveness and attractiveness. Thus, there is an increased interest in better integrating major fixed-route transit line with feeder lines.

Public transportation services have historically been categorized depending on their operating policy as either Fixed-Route Transit (FRT) or Demand-Responsive Transit (DRT). The growing broad category of "flexible" transit services includes all types of hybrid services that combine pure DRT services and FRT services. These services have established stop locations and/or established schedules, combined with some degree of demand-responsive operations. However, their use has been quite limited in practice so far.

The Demand-Responsive Connector (DRC) is considered as a flexible transit service because it operates in a demand-responsive fashion within a service area and moves customers to/from a transfer point that connects to the major transit network, so that it is formally defined as a "one-to-many" and/or "many-to-one" type of service. Koffman (2004) found that the DRC transit service has been operating in quite a few cities and is one of the most often used types of flexible transit services, especially within low density residential areas. However, feeder lines can also follow a traditional FRT policy, when demand is higher.

In designing such integrated transit systems for large communities, planners may divide the whole residential service area into zones for easier management of the operation, to reduce operating cost, and to provide a better level of service to customers. For instance, in Denver a system called "call-n-ride" provides demand-responsive services in zones connected to stations of a light-rail system. However, a non-optimal structure might often be adopted and sometimes there is a lack of zone design, because these services are still considered a niche market. Recent development of the modern society are suggesting to take a closer and better look at these type of services, which appear to be destined to represent an increasingly significant mean of transportation.

Research specifically on the DRC system is still quite limited. Cayford and Yim (2004) surveyed the customer demand for DRC for the city of Millbrae. They also designed and implemented an automated system used for DRC services. The service uses an automated phone-in-system for reservations, computerized dispatching over a wireless communication channel to the bus driver and an automated callback system for customer notifications. Yim and Ceder (2006) surveyed customers of Bay Area Rapid Transit and designed routing strategies.

Other types of flexible transit services include Mobility Allowance Shuttle Transit (MAST) services, also known as Route Deviation. Quadrifoglio et al. (2006, 2007, 2008a) have developed analytical models and heuristic algorithms to help design and schedule MAST services. Daganzo (1984) describes a flexible system in which the pick-up and drop-off points are concentrated at centralized locations called checkpoints. Cortés and Jayakrishnan (2002) proposed and simulated one type of flexible transit called High-Coverage Point-to-Point Transit (HCPPT), which requires the availability of a large number of transit vehicles. Aldaihani et al. (2004) developed an analytical model that aids decision makers in designing a hybrid grid network that integrates a flexible demand responsive service with a fixed route service. Häll et al. (2009) consider an integrated dial-a-ride Problem (IDARP) for designing vehicle routes and schedules for a dial-aride service where some part of each request may be performed by a fixed route service. The fixed routes are assumed to be given and it concerns how they are used for the DRT.

Purely DRT systems have been instead extensively studied. Savelsbergh and Sol (1995), Desaulniers et al. (2000) and Cordeau and Laporte (2003) provide comprehensive reviews on the proposed methodologies and solutions to deal with these very difficult problems. Some recent examples of research on DRT include survey of customer demand (Khattak and Yim 2004), zoning strategy (Dessouky et al. 2005), environmental impact (Dessouky et al. 2003), serviceability index (Sandlin and Anderson 2004), and fleet size (Diana et al. 2006).

Authors of this paper have recently been conducting research specifically on the DRC to fill the literature gap with respect to its design. Quadrifoglio and Li (2009) developed a model to determine the best operating policy to be adopted in one residential zone to maximize the level of service; they defined and derived the "critical demand density" representing the switching point between the fixed and demand responsive competing policies. A related validating application is also shown in Li and Quadrifoglio (2010). Finally, Li and Quadrifoglio (2009) developed a model to determine the optimal number of zones for the feeder transit systems, for which a one-vehicle operation is adopted in each zone.

This paper builds on this recent work and develops a model to determine the optimal number of zones in which a two-vehicle feeder transit operation is needed in each zone, when the demand is high enough and the distance between the farthest customer and the transfer terminal is long, often the case for modern sprawled residential large areas. The main purpose is to develop and provide tools using continuous approximation to guide planners in their design decisions with as little information as possible for planning out residential areas to maximize transit performance. While this type of modeling has its own limitations given by its inherent assumptions, these tools are beneficial for suggesting the best design of these services.

2 System description

In this section we describe our model representing a large residential area connected to a major transit line by feeder services, which can be following a fixed route policy (FRT) or operates as a DRC service. The objective is to describe the system in order to later develop an approximate but realistic analytical model to suggest the most appropriate zone design of the area and most appropriate operating policy depending on the circumstances and the value of the main parameters. Ultimately, we would like to suggest the appropriate number of zones in which the area should be divided to maximize the overall transit performance.

2.1 Service area and demand

The service area is a representation of residential communities and is modeled as a rectangle of width W and length L (see Fig. 1). The rectangular service area is an approximation of similar shapes to get a closed-form solution, and this approximation was used in Daganzo (2004). While the rectangular area should realistically represent many practical situations, further research might be in need for other shape types. The service area is divided into n zones with length L and width W/n. Within each zone the terminal connecting with the outside fixed-route major transit network is located at the half width on the far left of the zone, as we assume the modeled residential area is on one side of a major road. If both sides are developed equally, our analysis would be symmetrically doubled on the other side of the major road.

The spatial distribution of the demand, either a departure to (pick-up) or an arrival from (drop-off) the terminal, has a uniform distribution within the service area. The temporal distribution of the demand is assumed to be a Poisson process with constant average arrival rate λ for the whole service area. We assume that a fraction α of the customers need to be transferred from the service area to a major attraction destination (such as a city's downtown) through the terminals (pick-up customers) and a fraction $1-\alpha$ of them vice versa (drop-off customers).

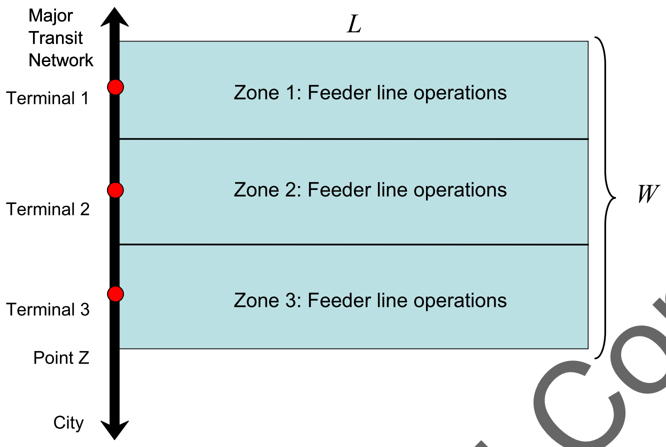

Fig. 1 Feeder line service area with three zones

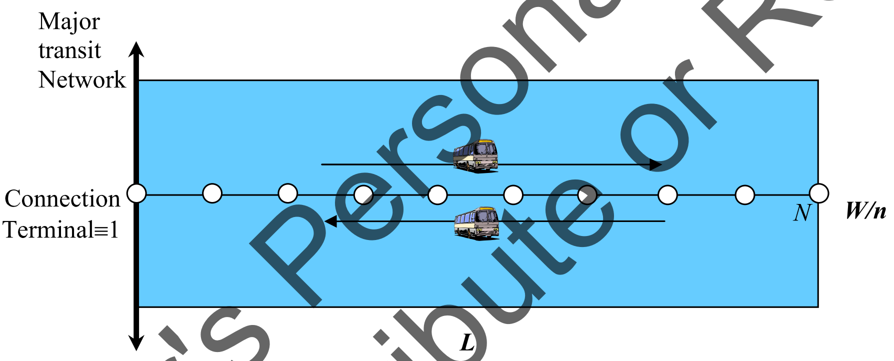

Fig. 2 FRT service in one zone

2.2 Transit operation policies

As shown in Fig. 1, the major fixed-route transit service connects terminals and transfers customers from the service area to the city or vice versa.

Within each service zone, a FRT policy or a DRC policy would be adopted to operate the feeder service. For each operating policy we consider only two vehicles moving at average speed v_b miles/hr and stopping at each station for a period of s.

2.2.1 FRT policy

Shown in Fig. 2, within each zone the FRT policy offers continuous service with the vehicle moving back and forth along the route between bus station 1 (connection terminal) and station N (located at the middle of the right boundary of the service area). There are $N-2$ stations between 1 and N and the distance between adjacent stations is a constant d miles. We assume that the two buses begin their operations at the same time leaving from stop 1 and N respectively. At any point in time during the operations, the vehicle moving left-to-right performs the drop-off operations (transferring customers from terminal 1 to the stops closest to their final destination) and the vehicle moving right-to-left performs the pick-up operations (transferring customers from their stops closest to their origin to terminal 1). The pick-up customers show up at random within the service zone, walk to the nearest station and wait for the bus. The drop-off customers show up and wait at the terminal, take a ride and then walk to their final destination at random within the service zone.

2.2.2 *DRC policy*

Within each zone the DRC policy provides a demand responsive terminal-to-door (and vice versa) service to customers. We assume that pick-up customers are able to notify their presence by means of a phone or internet booking service. The vehicle begins and ends each of its trips at the terminal. Immediately before the beginning of each trip, waiting customers (both pick-up and drop-off ones) are scheduled and the route for the trip in the service zone is constructed. We assume no "real-time" scheduling: customers requesting service while the DRC vehicle is performing a trip are scheduled and served in the following trip. Dynamic operations are certainly possible and improve the DRC service performance, if wisely implemented. However, our assumption is justified by the need to develop (approximate) analytical equations for the reminder of the paper to derive an optimal number of zones. It is very difficult to derive analytical formulas for real-time scheduling (this option can be considered for a further improvement of the model). We also feel that our simplification is quite reasonable since in practice most customers of demand responsive services are scheduled before the trip operations. This is particularly true when cycle times are relatively short and for multiple vehicle operations. As an example, SuperShuttle operates with a close to 100% pre-booking policy (de Neufville 2006). Paratransit services are also similar. In addition, should we be able to model a dynamic service, results would likely change very little in terms of optimal zone design. To schedule the requests we assume that the schedule is calculated by an insertion algorithm attempting to minimize the total distance traveled by the vehicle, as practically adopted by most transit agencies. Rectilinear movements are assumed since they better estimate distances traveled in real road networks and generally provide good approximations (see Quadrifoglio et al. 2008b).

We divide each service zone into two subzones with width W/n and length $L/2$ (see Fig. 3). Subzone 1 is adjacent to the terminal and Subzone 2 on the right of it. Each vehicle serves a subzone and vehicles operate continuously and alternate their operations among subzones, so their expected average cycle time is the same. This means that a vehicle would start from terminal 1, schedule its service for Subzone 1, serve Subzone 1 (while the other vehicle is serving Subzone 2), come back to terminal 1, board waiting drop-off customers for Subzone 2, move to Subzone 2, schedule its service for Subzone 2, serve Subzone 2 (while the other vehicle is serving Subzone 1) and come back to terminal 1.

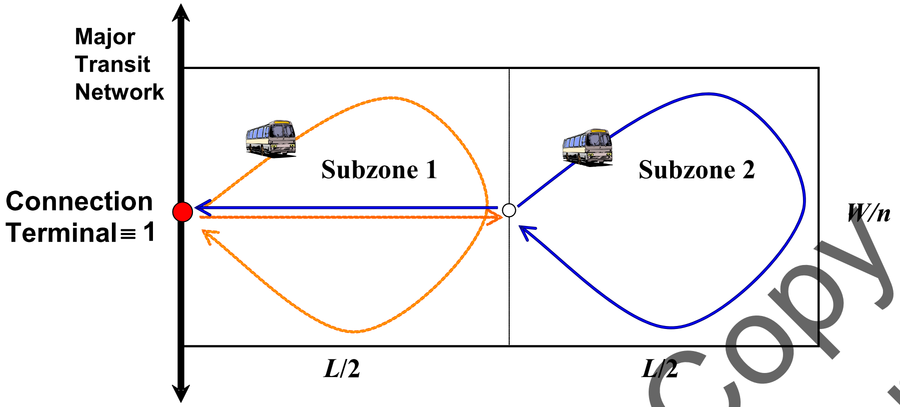

Fig. 3 DRC service in one zone

3 Analytical model

We next describe the development of the analytical model needed to determine the optimal number of bus stations N for the FRT policy and the optimal number of zones n for both FRT and DRC polices.

3.1 Total cost function definition

For the FRT policy a customer can be in the following states: walking between the house and the nearest bus station, waiting for the FRT, riding the FRT, waiting for the major transit, and riding the major transit. For the DRC policy a customer can be in states of waiting for an on-demand vehicle, riding an on-demand vehicle, waiting for the major transit, and riding the major transit. We assume that the different states of a customer may have a different cost to a customer.

The total cost of the designed system includes customer and vehicle cost. We have the following assumptions about total cost function definition:

- The vehicle cost of the major transit is not counted to determine the optimal n. The vehicle cost of the major transit is dependent on the number of vehicles determined by the headway and customer demand, but not n.
- The customer waiting time for the major transit is not included in the total cost definition. Since the major transit is assumed to have a constant headway, the customer waiting time for the major transit is independent on n. For drop-off customers, this waiting time is also independent on the FRT or DRC policy in a zone.
- We assume no coordination between the major transit headway and the FRT headway in a zone, the expected waiting time of pick-up customers at a terminal for the FRT policy is approximately the same as that for the DRC policy. This assumption is reasonable when assuming a high frequency service in the major transit line and/or when the major transit lines are multiple.

Then the total cost of the system for FRT policy and DRC policy are as follows. For pick-up customers waiting at home, the waiting time is counted from the show-up time, which is the moment that a customer is ready to be pick-up. If they book a ride, they will specify a pick-up time, from which the waiting time is counted.

$$\begin{aligned}\text{FRT Total Cost} &= \text{Customer Cost} + \text{FRT Bus Cost}\\ &= n\frac{\lambda}{n}\alpha\left\{a_k E\left(T_{wk}\right) + a_w E\left(T_{wt}^p\right) + a_b E\left(T_{rd}^p\right) + a_B E\left(T_{rd-B}^p\right)\right\}\\ &\quad + n\frac{\lambda}{n}(1-\alpha)\left\{a_B E\left(T_{rd-B}^d\right) + a_w E\left(T_{wt}^d\right) + a_b E\left(T_{rd}^d\right)\right.\\ &\quad \left. + a_k E\left(T_{wk}\right)\right\} + 2nF_b, \end{aligned} \tag{1}$$

$$\begin{aligned}\text{DRC Total Cost} &= \text{Customer Cost} + \text{DRC Vehicle Cost}\\ &= n\frac{\lambda}{n}\alpha\left\{a_w^h E\left(T_{wt}^p\right) + a_v E\left(T_{rd}^p\right) + a_B E\left(T_{rd-B}^p\right)\right\}\\ &\quad + n\frac{\lambda}{n}(1-\alpha)\left\{a_B E\left(T_{rd-B}^d\right) + a_w E\left(T_{wt}^d\right) + a_v E\left(T_{rd}^d\right)\right\}\\ &\quad + 2nF_v. \end{aligned} \tag{2}$$

3.2 Derivation of the computed variables in the total cost function

For the FRT and DRC policies the customers have the same ride time on the major transit. Shown in Fig. 1, in the service area, Z is the point nearest to the city. We define the ride time as the vehicle dwelling time plus vehicle running time between a terminal and Point Z. For customers transferring at terminal $k \in \{1, 2, \ldots, n\}$, the ride time is $(k-\frac{1}{2})\frac{W}{nv_B} + kS$. Then we have the following results for customer ride time on the major transit:

$$E\left(T_{rd-B}^p\right) = E\left(T_{rd-B}^d\right) = \frac{1}{n}\sum_{k=1}^{n}\left[\left(k-\frac{1}{2}\right)\frac{W}{nv_B} + kS\right] = \frac{W}{2v_B} + \frac{n+1}{2}S. \tag{3}$$

3.2.1 *FRT policy*

The width of each zone is W/n. According to Quadrifoglio and Li (2008), we have the following results for the FRT policy. In one zone, the expected walking time to the nearest bus stop $E(T_{wk})$ is

$$E\left(T_{wk}\right) = \frac{1}{4v_{wk}}\left(\frac{L}{N-1} + \frac{W}{n}\right); \tag{4}$$

the expected ride time of all customers is

$$E\left(T_{rd}^p\right) = E\left(T_{rd}^d\right) = \frac{1}{2}\left[\frac{L}{v_b} + (N-1)\,s_f\right] \tag{5}$$

and the expected waiting time of all customers is

$$E\left(T_{wt}^p\right) = E\left(T_{wt}^d\right) = \frac{L}{2v_b}\left[1 - \frac{1}{2(N-1)}\right] + \frac{s_f}{2}\left(N - \frac{3}{2}\right). \tag{6}$$

3.2.2 DRC policy

We approximate the insertion heuristic operations with a no-backtracking policy left-to-right on the top half and right-to-left on the bottom half of each subzone, as suggested by Daganzo (2004). C is the expected cycle time for each vehicle to serve both subzones of the whole service area. As shown in Fig. 3, vehicles alternate their operations between zones.

During a cycle C, both vehicles will visit each subzone, so pick-up customers will need to wait an average of $C/4$ from their show-up time, which is assumed to be the ready time for pickup, to the beginning of the operations of either vehicle within their zone, plus an additional average of $(C - L/v_b)/4$, a fourth of the cycle reduced by L/v_b, which is the total transfer time needed by each vehicle to switch zone, during which there are no pick-up nor drop-off operations. Thus, we have

$$E\left(T_{wt}^{p}\right) = C/2 - L/\left(4v_b\right), \tag{7}$$

and drop-off customers will need to wait an average

$$E\left(T_{wt}^{d}\right) = C/4. \tag{8}$$

The expected ride time in Subzone 1 is $E(T_{rd}^{p-1}) = E(T_{rd}^{d-1}) = (C - L/v_b)/4$, again a fourth of the cycle reduced by the transfer time L/v_b. The expected ride time for customers in Subzone 2 is instead $E(T_{rd}^{p-2}) = E(T_{rd}^{d-2}) = (C - L/v_b)/4 + L/(2v_b)$, since they need to spend onboard, in addition, the transfer time $L/(2v_b)$ to ride between the connection terminal and Subzone 2. Thus, we have that

$$E\left(T_{rd}^{p}\right) = E\left(T_{rd}^{d}\right) = \left[E\left(T_{rd}^{p-1}\right) + E\left(T_{rd}^{p-2}\right)\right]/2 = C/4. \tag{9}$$

Since the scheduling of customers is a vehicle routing problem, it is difficult to derive C analytically. Approximating the commonly used insertion heuristic scheduling procedure with a no-backtracking policy, Quadrifoglio and Li (2009) derived an analytical solution of C for the case of one zone. For each zone with demand λ/n and width W/n, C is the solution of a quadratic equation and is

$$C = \frac{-b - (b^2 - 4ac)^{1/2}}{2a}, \tag{10}$$

where:

$$a = \frac{\lambda}{4n}\left[\frac{\lambda}{n}\left(\frac{W}{6n} + s v_b\right) - 2v_b\right], \tag{11}$$

$$b = \frac{\lambda}{n}\left(\frac{5W}{6n} + \frac{3L}{2} + 2s v_b\right) - 2v_b, \tag{12}$$

$$c = 2L + \frac{8W}{3n} + 4s v_b. \tag{13}$$

The following condition should be satisfied to have $C > 0$:

$$n > \frac{1}{4}\left\{\lambda s + \left[(\lambda s)^2 + \frac{4\lambda W}{3v_b}\right]^{1/2}\right\}. \tag{14}$$

However, a closed-form expression for C is not easy to derive. Let k represent the average number of customers for a cycle time. Assume $k/(k+2) = 1$ which is true when $k \to \infty$. According to Quadrifoglio and Li (2009) we obtain a closed-form expression for the approximate cycle time, $\tilde{C}$, for each zone with demand λ/n and width W/n.

$$\tilde{C} = \frac{2sv_b + \frac{4W}{3n} + 3L}{v_b - \frac{\lambda}{2n}\left(\frac{W}{6n} + sv_b\right)}, \tag{15}$$

where n should satisfy expression (14) to guarantee $\tilde{C} > 0$.

3.3 Optimal number of zones

3.3.1 FRT policy

We substitute the computed variables in (1) and obtain the FRT Total Cost as

$$\begin{aligned} f(n, N) = {} & \frac{\lambda a_k}{4v_{wk}}\left(\frac{L}{N-1} + \frac{W}{n}\right) \\ & + \frac{\lambda a_w}{2}\left\{\left[1 - \frac{1}{2(N-1)}\right]\frac{L}{v_b} + \left(N - \frac{3}{2}\right)s_f\right\} + \frac{\lambda a_b}{2}\left[\frac{L}{v_b} + (N-1)s_f\right] \\ & + \lambda a_B\left(\frac{W}{2v_B} + \frac{n+1}{2}S\right) + 2nF_b. \end{aligned} \tag{16}$$

Although n and N are discrete variables, we assume they are continuous variables to derive the optimum. The partial derivative and the second order partial derivative of function $f(n, N)$ with respect to n and N are

$$\frac{\partial f(n, N)}{\partial n} = -\frac{\lambda a_k W}{4v_{wk}n^2} + \frac{1}{2}\lambda a_B S + 2F_b, \tag{17}$$

$$\frac{\partial^2 f(n, N)}{\partial n^2} = \frac{\lambda a_k W}{2v_{wk}n^3} > 0, \tag{18}$$

$$\frac{\partial f(n, N)}{\partial N} = -\frac{\lambda a_k L}{4v_{wk}(N-1)^2} + \frac{\lambda a_w}{2}\left[\frac{L}{2v_b(N-1)^2} + s_f\right] + \frac{\lambda a_b s_f}{2}, \tag{19}$$

$$\frac{\partial^2 f(n, N)}{\partial N^2} = \frac{\lambda L}{2(N-1)^3}\left[\frac{a_k}{v_{wk}} - \frac{a_w}{v_b}\right] > 0. \tag{20}$$

Since $\frac{\partial^2 f(n,N)}{\partial n^2} > 0$ and $\frac{\partial^2 f(n,N)}{\partial N^2} > 0$, the FRT total cost $f(n, N)$ is a convex function, and has a global minimum. When $\frac{\partial f(n,N)}{\partial n} = 0$, the optimal n value is

$$n = \left[\frac{\lambda a_k W}{2v_{wk}\left(\lambda a_B S + 4F_b\right)}\right]^{1/2}. \tag{21}$$

When $\frac{\partial f(n,N)}{\partial N} = 0$, the optimal N value is

$$N = 1 + \left[\frac{L}{2s_f\left(a_b + a_w\right)}\left(\frac{a_k}{v_{wk}} - \frac{a_w}{v_b}\right)\right]^{1/2}. \tag{22}$$

If optimal n is not an integer, the optimal integer number of zones is, because of convexity, either $\lceil n \rceil$ or $\lfloor n \rfloor$, whichever has the minimum total cost. Analogously, the optimal integer number of bus stations is either $\lceil N \rceil$ or $\lfloor N \rfloor$ with the minimum cost.

3.3.2 DRC policy

We substitute the computed variables in (2) and obtain the analytical rigorous DRC total cost $r(n)$ and its derivative as

$$\begin{aligned} r(n) &= \frac{\lambda}{2}\left[\alpha a_w^h + (1-\alpha)\frac{a_w}{2} + \frac{a_v}{2}\right]C \\ &\quad + \lambda\left[a_B\left(\frac{W}{2v_B} + \frac{n+1}{2}S\right) - \frac{\alpha a_w^h L}{4v_b}\right] + 2nF_v, \end{aligned} \tag{23}$$

$$\begin{aligned} \frac{dr(n)}{dn} &= \frac{1}{2}\lambda a_B S + 2F_v + \frac{\lambda}{2}\left[\alpha a_w^h + (1-\alpha)\frac{a_w}{2} + \frac{a_v}{2}\right] \\ &\quad \times \frac{\frac{\lambda}{2}\left[\frac{\lambda}{n}\left(\frac{W}{4n} + sv_b\right) - v_b\right]C^2 + \lambda\left[\frac{5W}{3n} + \frac{3L}{2} + 2sv_b\right]C + \frac{8W}{3}}{(2aC+b)n^2}, \end{aligned} \tag{24}$$

where C, a, b are obtained from (10) to (12).

In (23) we substitute C with the approximation $\tilde{C}$ from (15) and we obtain the approximate analytical DRC total cost $p(n)$ and its derivative as

$$\begin{aligned} p(n) &= \frac{\lambda}{2}\left[\alpha a_w^h + (1-\alpha)\frac{a_w}{2} + \frac{a_v}{2}\right]\left[\frac{2sv_b + \frac{4W}{3n} + 3L}{v_b - \frac{\lambda}{2n}\left(\frac{W}{6n} + sv_b\right)}\right] \\ &\quad + \lambda\left[a_B\left(\frac{W}{2v_B} + \frac{n+1}{2}S\right) - \frac{\alpha a_w^h L}{4v_b}\right] + 2nF_v, \end{aligned} \tag{25}$$

$$\begin{aligned} \frac{dp(n)}{dn} &= \frac{1}{2}\lambda a_B S + 2F_v - \frac{\lambda}{2}\left[\alpha a_w^h + (1-\alpha)\frac{a_w}{2} + \frac{a_v}{2}\right] \\ &\quad \times \frac{n^2 v_b(\lambda s^2 v_b + 4W/3 + 3\lambda Ls/2) + n\lambda W(sv_b/3 + L/2) + \lambda W^2/9}{[n^2 v_b - (\lambda/2)(W/6 + nsv_b)]^2}. \end{aligned} \tag{26}$$

Because of their convexity, when $\frac{dr(n)}{dn} = 0$ or $\frac{dp(n)}{dn} = 0$ the rigorous DRC total cost or the approximated DRC total cost have global minimum values. The corresponding optimal n has no closed-form expression, but it is possible to identify the optimal n numerically (or graphically) by plugging increasing n integer values into $r(n)$ and $p(n)$ and choose the ones with the minimum total cost.

4 Computational experiment

In this section, we demonstrate the capabilities of the analytical model in determining the optimal number of zones and the optimal number of bus stations. We performed simulation experiment to validate the DRC analytical modeling results. Our simulation model is described briefly here.

Step 1. Input number of zones n, customer demand λ, service area length L and width W. Generate each customer's show-up time and locations according to customer demand's temporal Poisson process and spatial uniform distribution.
Step 2. Increase time from 0 to 1000 minutes by 0.5-minute interval, and perform computations at time of scheduling. There are four types of scheduling: schedule Vehicle-1 customers in Subzone 1, schedule Vehicle-1 customers in Subzone 2, schedule Vehicle-2 customers in Subzone 1, and schedule Vehicle-2 customers in Subzone 2. The computations performed include creating the customer pick-up/drop-off sequence with an insertion heuristic algorithm, and computing customers' waiting time and ride time.
Step 3. Compute total DRC cost with (1).

The insertion heuristic algorithm used in Step 2 is a widely used schedule algorithm for demand responsive services to create the customer sequence choosing the minimum additional distance at each insertion step; see details of this algorithm in Quadrifoglio and Li (2008). Rectilinear distances are used and we assume no "real-time" scheduling. For each number of zones n, we performed 30 simulation replications and the resulting 95% confidence half-intervals are about 1% of the mean for the simulated total cost.

If one were to consider the simulation, the analytical derivation of the customers' waiting time and ride time would be difficult to perform because of the embedded vehicle-routing problem and the dependency of the two vehicle's operation. Therefore the analytical model in the previous section assumes that the two vehicle's operation is independent and vehicles are not allowed to backtrack with respect to their primary forward direction to serve customers, which is an approximation of the simulation model.

We assume the parameter values are as those listed in Table 1. Assumed values are reasonable to our knowledge and mostly derived from previous work and used here as an illustrative example only. Values can of course be modified according to specific needs of planners (or researchers) or to test other scenarios.

For the FRT policy we obtained $n = 4.5$ from (21), and $N = 11.6$ from (22). By using (16), the total cost (3371.2 \$/hr) for $n = 4$ and $N = 12$ is the smallest among total costs for $n = 4$ or 5 and $N = 11$ or 12. So the integer optimal number of zones

Table 1 Parameter values

Parameter	Value	Unit
W	6	Miles
λ	200	Customers/hr
a_k	30	\$/customer/hr
F_b	100	\$/veh/hr
L	4	Miles
α	0.5	
a_w^h	10	\$/customer/hr
a_w	20	\$/customer/hr
a_v	10	\$/customer/hr
a_b	10	\$/customer/hr
a_B	10	\$/customer/hr
F_v	100	\$/veh/hr
v_{wk}	2	Miles/hr
v_b	20	Miles/hr
v_B	30	Miles/hr
s	0.008333	Hr
S	0.025	Hr

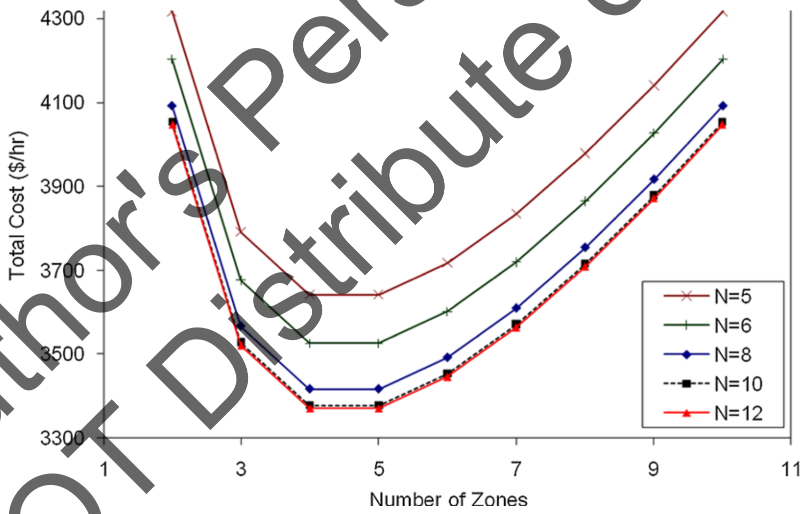

Fig. 4 FRT total cost for various n and N values

is 5 and the integer optimal number of bus stations is 12. The corresponding distance of adjacent stations is $4/(12-1) = 0.364$ miles = 1,920 ft, which is within the range [600 ÷ 2,500 ft] adopted by transit agencies in suburban areas (Texas Transportation Institute 1996). Figure 4 shows total FRT costs for various n and N values. The total cost is sensitive to the number of zones.

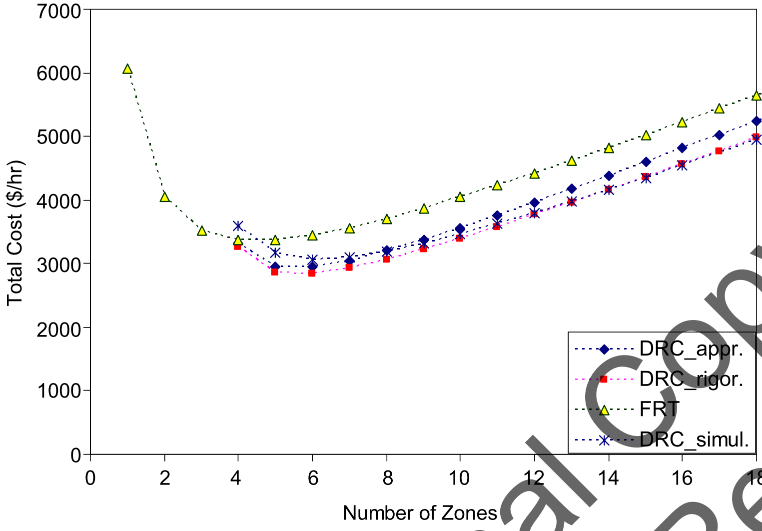

Fig. 5 Total cost functions for FRT and DRC policies

For the DRC policy, $n > 2.7$ with (14). By using the rigorous formulas, with (24) we obtained $n = 5.6$ when $\frac{dr(n)}{dn} = 0$; with (23), the total cost is 2867.2 \$/hr for $n = 5$ and is 2844.5 \$/hr for $n = 6$. So the integer optimal number of zones is 6.

By using the approximation formulas, with (26), we obtained $n = 5.5$ when $\frac{dp(n)}{dn} = 0$; with (25), the total cost is 2950 \$/hr for $n = 5$ and is 2944.2 \$/hr for $n = 6$. So the integer optimal number of zones is 6, the same as that with rigorous formulas.

The simulations show that the minimum total cost for the DRC policy with the insertion heuristic algorithm is 3060.7 \$/hr. The optimal number of zones is 6, which is the same as those from analytical rigorous and approximation formulas.

The total costs for various numbers of zones are shown in Fig. 5. We have the following observations:

- The minimum DRC total cost is less than that of the FRT policy, suggesting that the optimal configuration for this case would be a 6-zone DRC feeder policy.
- For the DRC policy, the total costs obtained from the approximation formulas, the rigorous formulas and the simulation are very close, validating the assumptions in our modeling approach.

For the DRC policy, the total cost obtained from simulations is slightly larger than that from rigorous formulas when $n < 9$; as expected (Quadrifoglio and Li 2009), this is caused by the existing correlations between the vehicles' operational cycles, which are not captured by our two-vehicle modeling, in which we assumed independency.

We would like to point out that the demand is highly variable in practice, generally following the traditional double peak pattern. Our methodology is able to identify a near optimal zoning design for every demand level. This could lead to suggest to dynamically modifying the zoning structure as a function of the demand. As this might not be a practical solution, so that the demand used for zone design might be the highest one at peak hour.

5 Conclusions

This paper addresses the optimal zone design problem faced by planners for feeder transit services with high demand levels and a large service area. The residential service area is modeled having a rectangular shape divided in zones and competing transit operations (demand responsive, DRC, and fixed route, FRT) are assumed to operate within each zone to connect residents to a major transit line through transfer terminals. Two vehicles are operating in each zone. Demand is assumed to have uniform spatial distribution and Poisson temporal distribution. An insertion heuristic algorithm operating within rectilinear movements is assumed to replicate the DRC operations. By balancing customer service quality and vehicle operating cost, we obtain closed-form and approximate expressions to determine the optimal number of zones and the best operating policy. Simulations are then used to validate the results of the analytical formulas and an illustrative example is shown.

Our model might only be useful for service areas close to a rectangular shape and service area with a uniform land-use pattern, which are, however, the majority of residential housing areas. Future research might possibly include applications of our approach to real case studies with collected demand data and actual road networks.

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