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Transit network design for small-medium size cities

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ABSTRACT

This paper proposes a novel heuristic to solve the network design problem for public transport in small-medium size cities. Such cities can be defined as those with a diameter of a few kilometers with up to a few hundred thousand residents. These urban centers present a specific spatial configuration affecting the land use and mobility system. Transportation demand is widespread in origin and concentrated in a small number of attraction points close to each other. This particular structure of demand ('many-to-few') suggests the need for specific methodologies for the design of a transit system at a network level. In this paper, such design methodologies are defined in terms of models and solution procedures and tested on a selected case study. The solution methods show promising results. The key variables of the model are the routes and their frequencies. The constraints of the problem affect the overall demand to be served, the quality of the proposed service (transfer, load factors) and the definition of routes.

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1. Introduction

Small-medium size cities can be defined as urban centers with a population of a few hundred thousand inhabitants and limited spatial dimensions (with a diameter of a few kilometers). These cities have specific characteristics in terms of land use and mobility system, which very often are similar to each other. These common characteristics, which generally come out from actions and choices layered over time, can be described as follows:

- a small historic center located in the central area of the city, often built-up in past centuries and with roads of limited width. This area contains the main part of city activities: the local administration and other various services, schools, hospitals, commercial and service activities. Thus, this area is characterized by the presence of both residences and jobs;
- a radial structure for the other neighborhoods, generally more recently developed, which extend along the main roads connecting other nearby towns. These districts are reserved almost exclusively for residential use with a limited presence of work activities. Building development is related to the distance from the center: the farther away, the less densely built-up; and

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- important productive (industrial) areas and commercial areas (shopping centers) may also be present at the periphery.

The urban structure described above affects the mobility system, which is typically characterized by the following features:

- transport demand origins are widespread;
- transport demand attractions are instead concentrated at a few major points, often located in the city center, such as administrative centers, school campuses, railway station, hospital and production areas;
- the road network is radially shaped and has relatively few connections; and
- public transport is operated by buses without any hierarchy among different lines (i.e. main and feeder lines).

Such a typical 'many-to-few' structure of mobility demand as well as the low connectivity of the road network suggests conceiving ad hoc models for transit network design, tailored to this peculiar spatial configuration. Such peculiar but fairly common structuring of the data is the driving criterion for developing the proposed design method. As an example, in a small size town where trips last no more than a 10–20 minutes, transit transfers are hardly tolerable for users and should be reduced as much as possible. Dealing with such expectations of transit users and including them in the design approach represents an important innovation with respect to other methods proposed in literature. In general, the innovative capability of these heuristics is to explicitly consider and control relevant issues involving the quality of the design (i.e. transfers, overlapping routes, satisfied demand, route directness, etc.).

The paper is structured as follows: the next section presents the state-of-the-art review on the transit network design problem; the third section describes the mathematical formulation of the optimization problem; the fourth section illustrates the solution procedure; the fifth section presents the results of the application of the procedure to a real-life network; and the final section offers some concluding remarks.

2. Background

The transit network design (TND) problem is a well-known complex and non-convex problem (Newell 1979; Ceder and Wilson 1986; Baaj and Mahmassani 1991), usually formulated as a non-linear optimization problem with both discrete and continuous variables and constraints. The best and most efficient solution methods are based on heuristic procedures and meta-heuristic algorithms. A global review of route design, frequency setting, timetabling of transit lines, and their combination has been proposed by Desaulniers and Hickman (2007), Guihaire and Hao (2008), Kepaptsoglou and Karlaftis (2009) and Schöbel (2012). Many solution procedures and algorithms have been proposed in the literature. Some works are especially focused on the phase of route generation: Baaj and Mahmassani (1995) with an Artificial-Intelligent heuristic algorithm for route generation; Carrese and Gori (2002) with a heuristic procedure for large urban areas with different categories of lines; Bagloee and Ceder (2011) with a heuristic procedure providing routes categorized by hierarchy (mass, feeder, local routes); Beltran et al. (2009) with an innovative application for the generation of routes operated with green vehicles; Mauttone and Urquhart (2009) with a Pair Insertion Algorithm (PIA) inspired by the Route Generation Algorithm (RGA) of Baaj and Mahmassani, where its original expansion of routes by inserting individual vertices is replaced by a strategy of insertion of pairs of vertices; Michaelis and Schöbel (2009) with a heuristic based on a new approach integrating line planning, timetabling and vehicle scheduling; Cipriani, Gori, and Petrelli (2012) with a route generation procedure based on the flow concentration process and a parallel genetic algorithm for finding a sub-optimal set of routes with associated frequencies; and D'Acierno et al. (2014) with two methods for service redesign in the case of budget reductions. Some work is focused on the implementation of meta-heuristic algorithms: Pattnaik, Mohan, and Tom (1998) providing one of the first applications of genetic algorithms; Chakroborty (2003) working to highlight the effectiveness of using genetic algorithms for solving the urban TND problem; Kuan, Ong, and Ng (2006) proposing the design and the analysis of two metaheuristics (genetic algorithms and ant colony optimization) for solving the feeder bus network design problem; Fan and Mumford (2010) with an approach using hill-climbing and simulated annealing algorithms; Zhao and Zeng (2008) with a metaheuristic search scheme that combines simulated annealing, tabu and greedy search methods; Lownes and Machemehl (2010) providing a new mixed integer model for a single-route circulator design problem; Szeto and Wu (2011) proposing a genetic algorithm hybridized with a neighborhood search heuristic to tackle the frequency setting problem; and Yan et al. (2013) with a heuristic solution approach, based on k-shortest path algorithm, simulated annealing algorithm, Monte Carlo simulation, and probit-type discrete choice model. These procedures usually work to address the solution in a many-to-many context rather than to a more characterized environment, like that analyzed in this study for small-medium size cities. General transit network design procedures are able to solve the TND problem in any possible context but, in many cases, this positive aspect is more than counterbalanced by very complex solution methods, which may be inefficient in solving small problems with specific features.

3. Problem definition

In this paper, the transit network design (TND) problem is formulated as an optimization problem whose objective function, OF, accounts for the use of resources and impacts of transport on different stakeholders; that is, operators and users.

The formulation considers the total distance and time traveled by buses, which determines operator's variable costs; the total number of vehicles used, which determines operator's fixed costs; and the disutility of users, composed of in-vehicle travel time, waiting time at bus stops and perceived discomfort of transfers.

The objective function (OF) is therefore a weighted sum of these different transport cost items:

$$C_1 \sum_{i \in I_i} L_i \cdot f_i + C_2 \sum_{i \in I_i} \sum_{hk \in I_{a,i}} tp_{hk,i} \cdot f_i + C_3 nb + C_4 \sum_{i \in I_i} \sum_{hk \in I_{a,i}} tp_{hk} \cdot p_{hk,i} + C_5 \sum_{i \in I_i} \sum_{hk \in I_{a,i}} ta_{hk,i} \cdot pa_{hk,i} + C_6 \sum_{n \in I_n} nt_n, \quad (1)$$

where

$\sum_{i \in I_i} L_i \cdot f_i$ total distance traveled

$\sum_{i \in I_i} \sum_{hk \in I_{a,i}} tp_{hk,i} \cdot f_i$ total travel time

nb number of used buses

$\sum_{i \in I_i} \sum_{hk \in I_{a,i}} tp_{hk} \cdot p_{hk,i}$ total users' time in vehicle

$\sum_{i \in I_i} \sum_{hk \in I_{a,i}} ta_{hk,i} \cdot pa_{hk,i}$ total users' waiting time

$\sum_{n \in I_n} nt_n$ total number of transfers

The notations introduced in the formulae above are defined as follows:

L_i is the length of the line i (km), f_i is the frequency of line i (bus/h); I_i, $I_{a,i}$, I_n are respectively the set of lines, links and nodes; $tp_{hk,i}$, $ta_{hk,i}$ are respectively the travel time and waiting time on link hk of line i; $p_{hk,i}$, $pa_{hk,i}$ are respectively on board passengers and boarding passengers on link hk of line i; nt_n is the number of passengers transferring between any two lines on the node n; C_1, C_2, C_3, C_4, C_5 and C_6 are the monetary weights of the different items.

Input data include the origin-destination matrix (O/D), the road network, the bus capacity and the network terminals. Constraints follow general guidelines to make the problem practically fitting real-world instances. Specifically: the total amount of demand to serve; the desired quality of service (no more than one transfer for any O/D pair); all central attractors served from any zone without transfers; avoiding line circuits as well as too long or too short route lengths; maintaining the load factor lower than a predetermined threshold value. These will be formally defined in the solution procedure in the next section.

4. Solution procedure

As anticipated in the literature review, the proper formulation of the general TND problem is extremely cumbersome as it involves the simultaneous and combined solution of vehicle routing, assignment, facility location, line recombination and scheduling problems, many of which, even alone, would be challenging optimization problems, hardly solved in a reasonable time for realistic large instances. Consequently, solution procedures to such general formulations may be mainly an academic exercise, potentially with relevant insights, but usually with weak practical implications. Therefore, the proposed approach to solve the problem is a heuristic procedure. The goal is to provide a practical and fast algorithm able to offer solutions that are better than current practices for the urban areas with the characteristics described above.

The heuristic procedure can be summarized with the following steps:

- localization of central and peripheral terminals;
- construction of the base network;
- expansion of the base network;
- connection of lines of the base network with all central attractors;
- line frequency determination;
- linking of routes; and
- iterative adjustment of line frequencies.

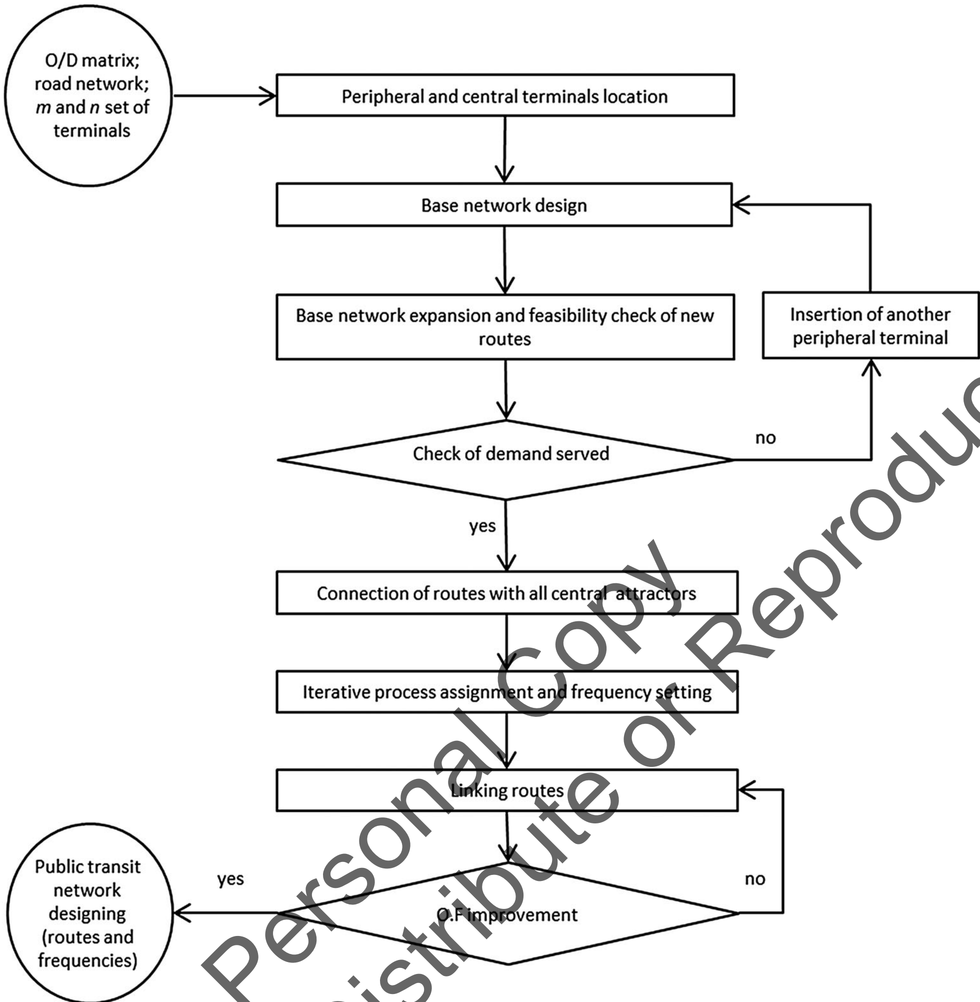

Figure 1. Solution procedure framework.

The procedure is depicted graphically in Figure 1 and detailed in the step-by-step description below.

Step 1 – Localization of terminals

First, terminals are located at the main nodes of attraction and generation. Let m be those situated in the central area and n those situated in the peripheral area. The choice of m and n is left to the planner and can be modified during the procedure.

The number of terminals is extremely important in terms of possible solutions generated – if it exceeds, respectively, 4–5 central terminals and 10–12 peripheral terminals, the large number of combinations between the peripheral and central terminals involves switching from a deterministic optimization procedure to a stochastic one.

Step 2 – Base network definition

The next step is to determine the base network, which is built by linking each peripheral terminal to the nearest central terminal trough the shortest path. The steps of the algorithm are listed as follows:

- select one of n external terminals;
- generate m paths from the selected peripheral terminal to each of the central ones, finding the shortest paths in terms of travel time; and
- identify the shortest path among all the m generated ones.

The operation is repeated for all the external terminals (see Figure 2). The network composed by the set of n shortest paths connecting each peripheral terminal with the closest central one represents the most efficient network that satisfies the principal component of transport demand; that is, the demand originated in the major peripheral areas and directed to the city center.

Step 3 – Expansion of the base network
The base network must be expanded in order to increase the total amount of demand served. The first step is to assign the demand to the transport network according to an 'All-Or-Nothing' technique assignment. Then, a systematic procedure of path modifications is applied for each previously generated route (see Figure 4). Such a procedure consists of increasing the travel cost of a selected link by a predetermined value (e.g. +50%). Afterwards, the new shortest path between the endpoints of the link selected is computed and the feasibility of the new route is tested according to these criteria:

- the new route allows to serve demand not yet satisfied;
- the length increase is less than a predetermined threshold value (e.g. 50%); and
- the modification of the route is rational in terms of alignment; that is, each node has to be closer to the final destination than the previous one, even compared to other possible routes.

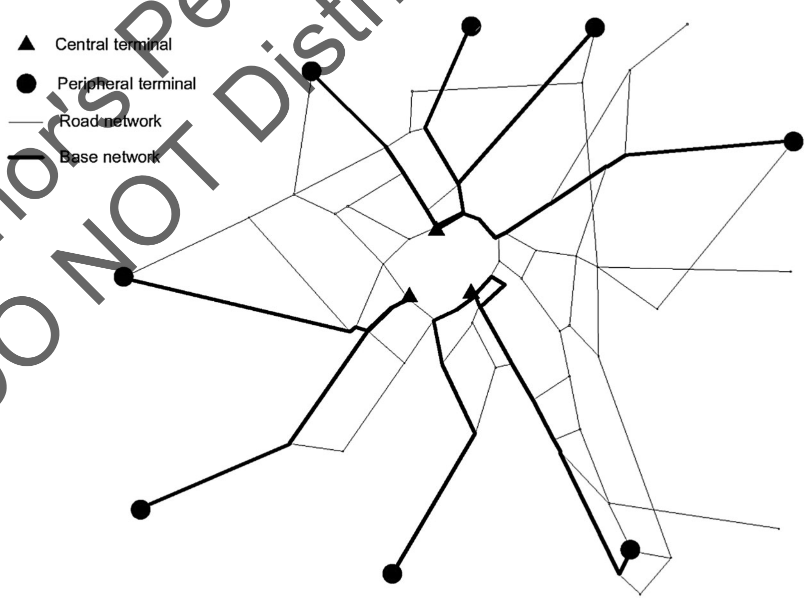

Figure 2. Terminal selection and base network definition.

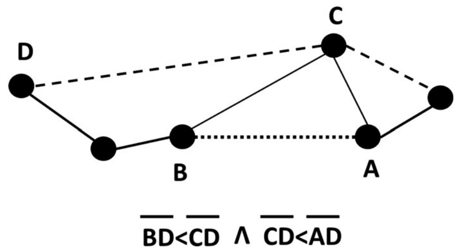

Figure 3. Replacement of link AB.

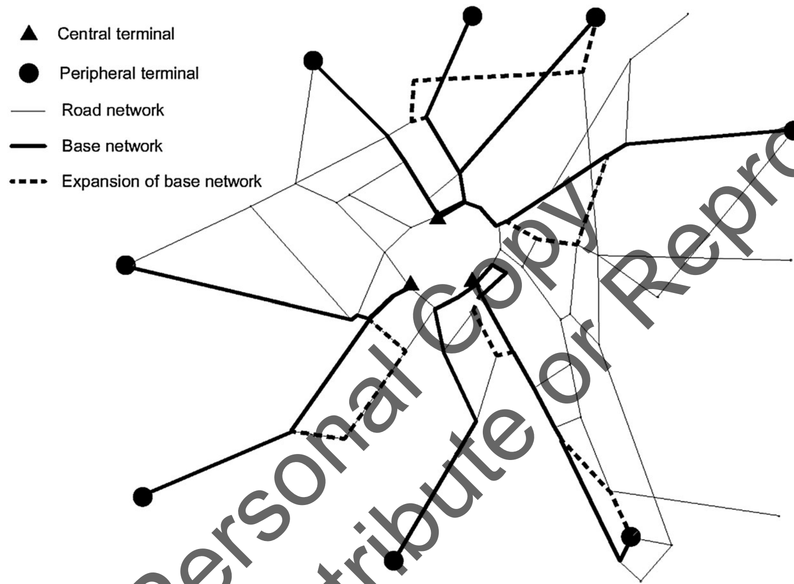

Figure 4. Expansion of base network.

The last criterion is equivalent to verify both the two conditions shown in Figure 3, which refer to the insertion of node C in the route between A and B. The insertion constraint is satisfied if: (a) the travel distance from B to D, final terminal of route, is less than that from C to D; and (b) the travel distance from C to D is less than that from A to D.

The increase of the travel cost of the selected link enables to preserve network connection. This condition could not be guaranteed if the link is cut, as often used in other works (cf. Mandl 1980; Pattnaik, Mohan, and Tom 1998).

There are several possible selection criteria for the choice of link to be possibly replaced. The two criteria described as follows are the prevalent: (1) proceed in a sequential way by analyzing, route by route, all the links belonging to the route starting from one of the two terminals and (2) sort all links of the routes by passenger load and select them, one at a time, from the lowest to the highest.

The second criterion seems to be more efficient from a computational point of view: in fact, deleting from the routes the links with the smallest level of passengers may lead to satisfy the minimum value of served demand before checking all the links of the network. In general, however, the topology of the network and the distribution of the demand are the fundamental references in the choice of the most suitable link selection rule.

At the end of the expansion phase of the base network (see Figure 4), the procedure checks if the served demand is greater than a fixed threshold x, for example, 95%, through the following relationship:

$$\sum_{hk \in I_a} pa_{hk} - \sum_{n \in I_n} nt_n \geq x \sum_{ij \in I_{OD}} s_{ij} \tag{2}$$

where s_{ij} are the trips between nodes i and j belonging to I_{OD} set.

If the constraint is not satisfied, a new iterative construction procedure is required by introducing and expanding additional new routes (see Figure 5). This phase is structured in the following steps:

- select a new additional peripheral terminal;
- increase the travel cost of the links of at least one route, by a predetermined factor, in order to reduce possible overlaps of routes; and
- generate the new route of the shortest path to a central terminal, and check the possible expansions.

These steps are repeated by adding new routes up to satisfy the constraint on the total amount of transport demand served.

Step 4 – Connecting routes to all central attractors

The connection of the routes to all central attractors consists in extending each route to all the other central attractors not reached by the route (see Figure 6). The extension takes

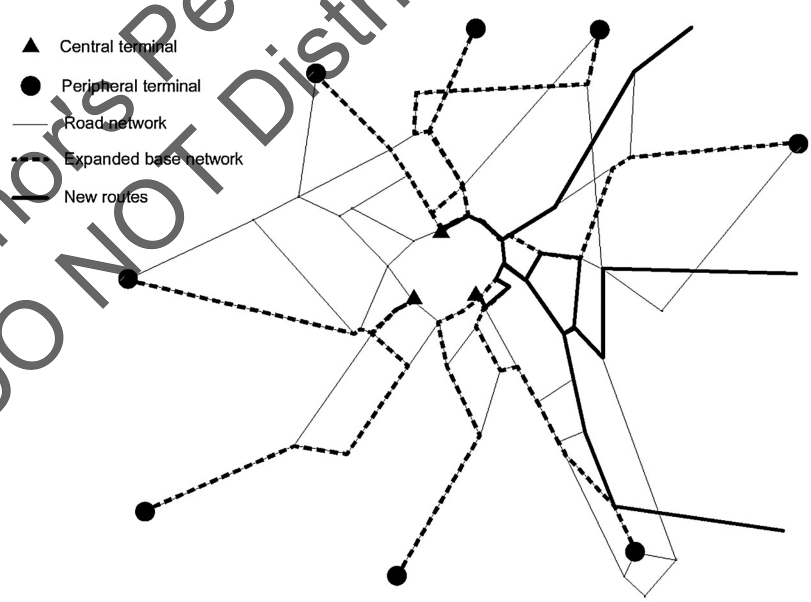

Figure 5. Generation of new routes.

place in the direction that corresponds to the minimum total travel time of passengers on route i:

$$\min \sum_{hk \in I_{a,i}} tp_{hk,i} \cdot p_{hk,i}. \tag{3}$$

The connection phase allows transfers from any route to any other one, automatically satisfying the constraint of not having more than one transfer to reach any possible destination. In addition, through this step, it is possible to reach directly, without transfers, all the major attractors from any external terminals.

Step 5 – Frequency setting

After the fourth step, a first feasible solution (the base network) has been built and will be improved in the following steps. First of all, an iterative assignment and frequency setting procedure, is applied to set the line frequency. The procedure starts by defining an initial set of frequencies. After the first assignment, based on the maximum load p_{max} recorded on each line, the frequency f_i is estimated by taking into account the maximum permitted load factor on board fc_{max} and the capacity of the vehicle C_V:

$$f_i = \frac{p_{\max}}{fc_{\max} \cdot C_V}. \tag{4}$$

The frequency thus obtained is then compared with the previous one. The process ends if there are no significant variations between two successive iterations; otherwise, a further assignment is required. The transport demand is loaded on the base network by means of a hyper-paths assignment procedure. The convergence of the iterative frequency setting

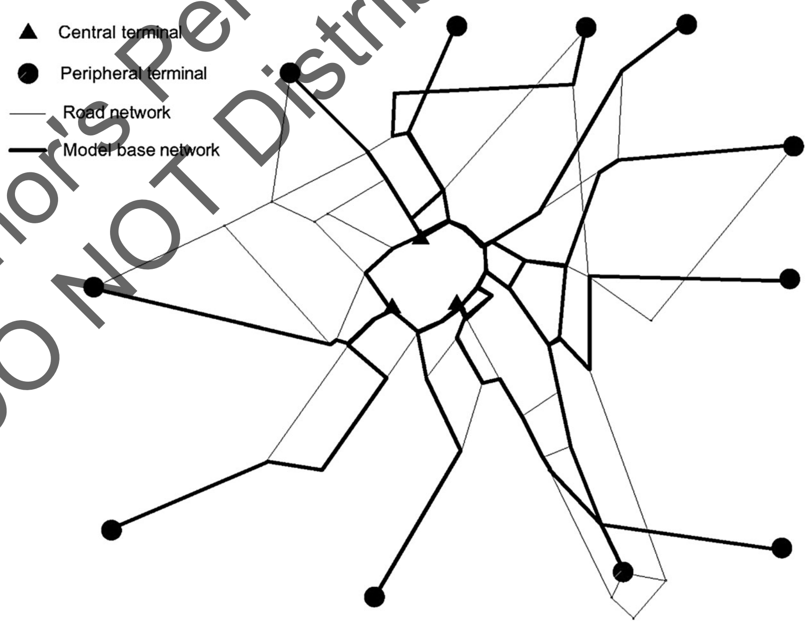

Figure 6. Connection of routes to all central attractors.

procedure is not guaranteed, but all computational tests performed converged in few iterations.

Step 6 – Linking of routes

To further reduce the number of transfers it is necessary to proceed to join routes found up to now and to move their central terminal from the center to the external terminals. This step should also allow to limit the overall length of the network, reducing the number of overlapping routes. The process of linking routes is not necessarily unique, since each route can be combined with more other routes.

Linking of routes is carried out through the following steps:

- select the line s with highest frequency;
- individuate the set of all paths of the passengers on the selected line;
- identify the lines to or from which transfers take place;
- for each of these lines, define the number of runs C_i according to the following criterion:

$$C_i = \left\lceil \frac{n_i}{\sum_{i \in I_{id}} n_i} \cdot C_s \right\rceil, \quad (5)$$

  where C_s, n_i, I_{id} represent, respectively, the number of runs of the selected line s, the number of transfers between the line s and the line i and the whole set of the identified lines;
- join the line s with all the lines belonging to the set I_{id} that present a number of runs C_i equal or greater than 1 (an example of the combination of two routes is represented in Figure 7);
- assign the O-D demand matrix, repeat the frequency setting procedure and compute the new OF value; and
- compare the OF value with that of the last solution and accept the linking of the routes if an improvement of the OF value is achieved.

The algorithm repeats the sequence of operations for the remaining lines; these are always selected in decreasing order of number of runs. The process ends when all the routes carried out in the design network have been processed.

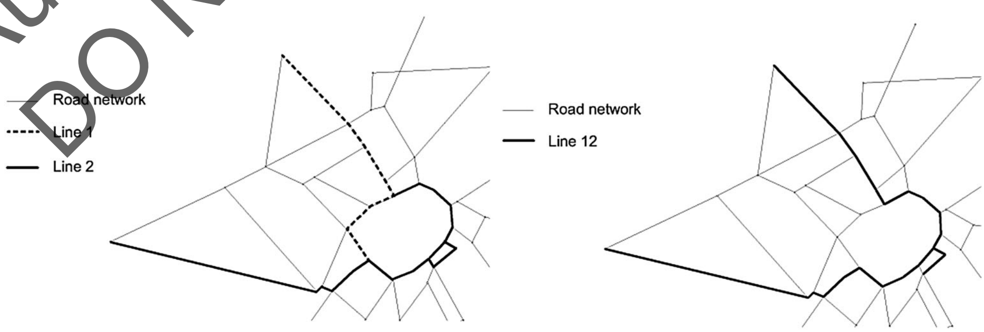

Figure 7. Linking routes of two lines.

Step 7 – Frequency adjustment
Finally, the frequencies of the lines are adjusted again applying the procedure described in Step 5, without any further route modification.

5. Case study

The proposed procedure has been applied to a real network of a small-medium size town, Foligno, an urban center of about 55,000 inhabitants located in the Umbria region of Italy. The aim of the application is to identify possible problems arising from the practical implementation of the procedure and to assess its performance through a comparison of the values of the objective function resulting by the application of the proposed model and those corresponding to the actual transit network. For the aim of the study, a transport model of the transit network is carried out. The urban area is divided into 30 traffic zones. The O/D transit matrix consists of approximately 3,130 trips in the peak hour. The historical city center and the external area with industrial activities represent the two major attraction areas and they are the destination for about two-thirds of the total transport demand (about 2,000 trips), while origins are distributed in all traffic zones.

The actual transit network consists of 16 bus lines with fixed routes and frequencies that connect the external areas, surrounding the center of Foligno, with the historical town center and the railway station; five lines pass through the city center.

Figure 8 illustrates the capability of the procedure to provide a good coverage of the urban area not only in terms of transport demand served but also in terms of links

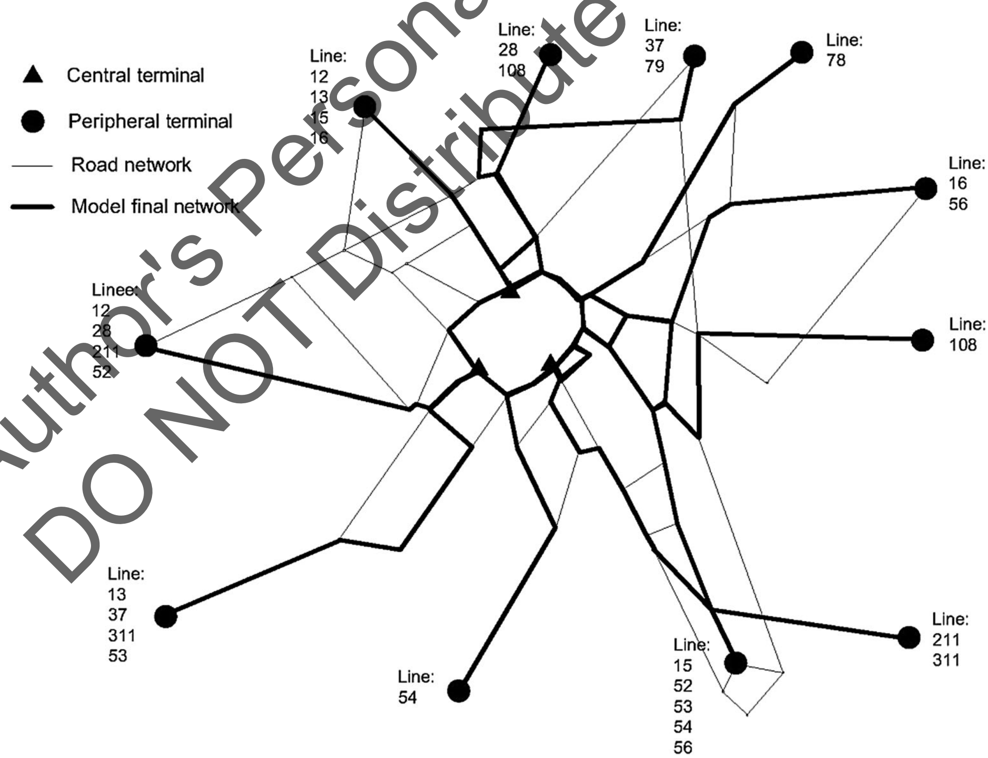

Figure 8. Final model network for the Foligno application.

Table 1. Performance comparison between existing network, base design network and final design network in the Foligno application.

Indicator	Existing network (a)	Base network (b)	Final network (c)	Δ (%) (b-c)	Δ (%) (c-a)
Number of lines	16	11	14	+21%	−12%
Buses-km	347	375	348	−7%	0%
Buses-h (min)	1,015	1,040	975	−6%	−4%
Number of buses	23	22	23	+4%	0%
Number of transfers	2,710	1,260	860	−32%	−68%
Total in-vehicle time (min)	33,940	32,625	32,935	+1%	−3%
Total waiting time (min)	39,460	67,640	34,310	−9%	−13%
Avg load factor	0.42	0.39	0.42	+8%	0%

belonging to the road network. It is also worth observing that the high number of lines around the central ring ensures both opportunity of transfers among every line and provides a high number of runs available for passengers traveling in the central areas of the city.

Table 1 shows the values of the different components of the OF, computed from the results of the simulations carried out both on the design network and on the existing one. Regarding the design network, the table shows the data related to both the initial configuration of the network (Step 4 – Connection line with all the main attractors) and the final configuration.

A detailed analysis of different components of the objective function highlights the importance of the final phase of line joining (Step 6), which eliminates several overlaps of lines that do not affect travelers paths and improves both travelers' and operator's cost. Moreover, the algorithm's solution attains to reduce the number of transfers (−32%), which are one of the most significant disutilities for travelers, affecting the onboard passengers' travel time only by a negligible quantity (+1%).

Results achieved by linking lines is convenient also from the operator's point of view, since the space and time distances traveled by vehicles in the final network are shorter than those of the base network as 7% and 6%, respectively.

Further considerations arise by the comparison of the actual (a) and the final design (c) transit network. With almost the same quantity of transport service supplied, the design network provides a qualitatively better service, as it reduces the number of transfers by 68%, the onboard passengers' travel time by 3% and the passengers' waiting time by 13%. It is worth mentioning that benefits for passengers are achieved without detriment for the operator, since the design network needs the same number of vehicles, the same total space distances and slightly lower time distances (−4%).

6. Conclusions

In this paper, we have developed a novel heuristic procedure that aims to provide a better solution for the transit network design problem for small-medium size cities, a large number of which exist throughout many European countries and all have similar spatial configuration, which drove our choices in developing and identifying the proper steps for the algorithm.

The proposed procedure provides encouraging results and is associated with a remarkable ease of execution on real networks. The Foligno case study shows significant improvements in nearly all considered performance measures when compared to current practices.

The ease of both the operation and the logic on which the algorithm is built allows for an easy understanding of the procedure which facilitates potential adjustments of the designer. The dependence on initial choices of parameters and constraints, typical of deterministic heuristics, is in fact partly offset by the possibility for the designer to intervene during the procedure run and correct some algorithm threshold, if necessary.

The heuristic is polynomial and suitable and capable to solve much larger size city problems, whose more complex structure and characteristics could, however, undermine the underlying assumptions applied to design transit networks in smaller cities. This would encourage further research on the topic to conveniently expand the proposed heuristic procedure to properly solve the TND of larger size cities with similar layout.

Disclosure statement

No potential conflict of interest was reported by the authors.

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