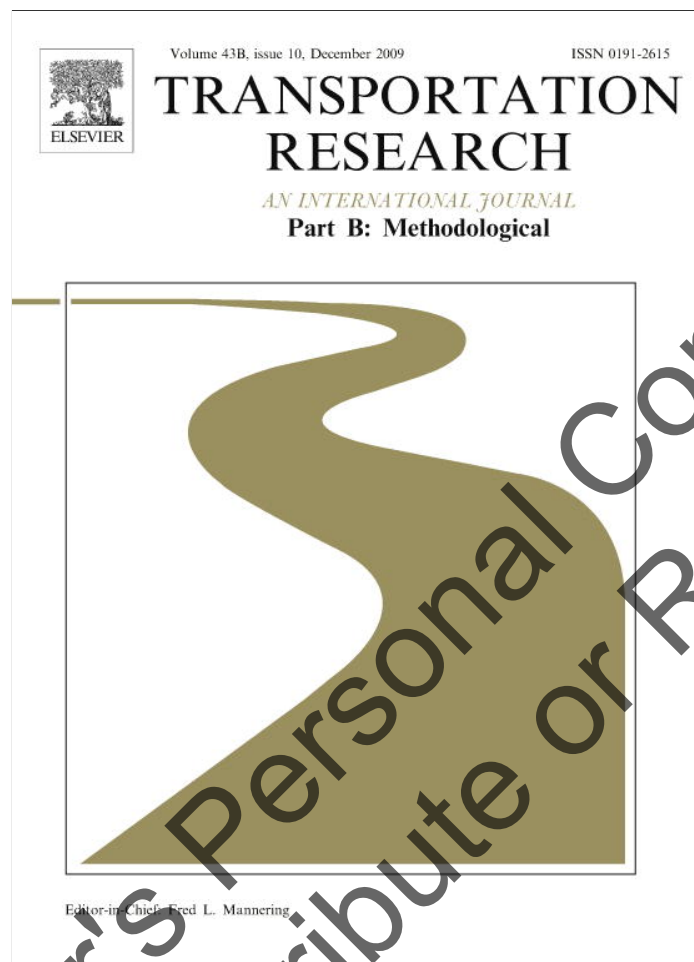


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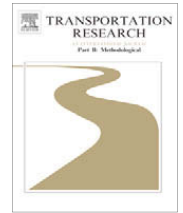
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Transportation Research Part B

journal homepage: www.elsevier.com/locate/trb

A methodology to derive the critical demand density for designing and operating feeder transit services

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ARTICLE INFO

Article history:

Received 30 April 2008

Received in revised form 19 March 2009

Accepted 13 April 2009

Keywords:

Feeder transit

Flexible transit

Demand responsive

Continuous approximation

Critical demand

ABSTRACT

Feeder lines are one of the most often used types of flexible transit services connecting a service area to a major transit network through a transfer point. They often switch operations between a demand responsive and a fixed-route policy. In designing and running such systems, the identification of the condition justifying the operating switch is often hard to properly evaluate. In this paper, we propose an analytical model and solution of the problem to assist decision makers and operators in their choice. By employing continuous approximations, we derive handy but powerful closed-form expressions to estimate the critical demand densities, representing the switching point between the competing operating policies. Based on the results of one-vehicle and two-vehicle operations for various scenarios, in comparison to values generated from simulation, we verify the validity of our analytical modeling approach.

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1. Introduction

Over the last few decades, modern urban areas, especially within residential communities, are experiencing a steady decrease in their population density as a consequence of urban sprawl, one of the most evident phenomena of our time. In the US, from 1960 to 2000, the population density dropped 15% despite an average overall population growth of 86% (www.demographia.com). In the majority of the rest of the world this trend is even more evident. This increasing “dispersion” of population causes conventional fixed-route transit systems serving those areas to become progressively more inefficient and relegated to a marginal role, since they are designed to serve few established routes and they heavily rely on concentrated demand.

Traditionally, transit services have been divided in two broad categories: fixed-route (FRT) and demand responsive (DRT). The typical cost efficiency of FRT systems is due to the predetermined schedule, the large loading capacity of the vehicles and the consolidation of many passenger trips onto a single vehicle (ridesharing). However, the general public considers them to be inconvenient because of their lack of flexibility, since often the locations of pick-up and/or drop-off points and/or the service's schedule do not match the individual rider's desires. Therefore, an increasingly larger portion of the growing population relies almost exclusively on private automobiles for their transportation needs, causing many urban areas to suffer from increasing congestion and pollution problems.

DRT systems instead provide much of the desired flexibility with a door-to-door type of service, but they are generally much more costly to deploy and, therefore, largely limited to specialized operations such as taxicabs, shuttle vans or

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dial-a-ride services, other than paratransit services (mandated under the ADA). Hence, transit agencies are facing a growing demand for improved and extended DRT services.

The broad and fairly new category of “flexible” transit services includes all types of hybrid services that combine pure demand responsive and fixed-route features. These services have established stop locations and/or established schedules, combined with some degree of demand responsive operation. Their characteristics have, in several cases, efficiently responded to some of the needs and wants of both the customers and the transit agency as well. However, their use has been quite limited in practice so far, as opposed to regular FRT systems.

The Demand Responsive Connector (DRC), also known as “feeder” transit line, is one type of flexible transit service. A survey conducted by Koffman (2004) for a Transit Cooperative Research Program (TCRP) project found that the DRC has been operating in quite a few cities and is one of the most often used types of flexible transit service, especially within low density residential areas. Examples can be found in Denver (CO), Raleigh (NC), Akron (OH), Tacoma (WA), Sarasota (FL), Portland (OR) and Winnipeg (Canada). The service operates in a demand responsive fashion within a service area and moves passengers from/to a transfer point that connects to a major fixed-route transit network, thus closing the gap perceived as the most critical by the majority of the potential transit users.

In most cases, the service operates as a FRT service during daytime and switches to a DRC type of service during evenings, nights or early morning, when the demand is lower. The customers know the policy in effect via a published schedule, telephone calls and/or the internet. When designing and operating such systems, planners need to decide what type of operations, between FRT or DRC, would be the most appropriate and/or what conditions would justify a “switch” from FRT to DRC (or vice versa). The decision is not straightforward, mainly because the demand for the service is often unknown beforehand and it will depend on the established service itself. In addition, even assuming a known demand, it is not clear what would be the best type of service. This is because the service quality provided to customers is not easy to assess and might depend on external conditions, such as safety, weather, time of the day; plus, the balance between operating costs and service quality is also frequently hard to evaluate.

With the ultimate goal of improving the efficiency and performance of these types of services, we present a methodology to assist decision makers in their choice by providing an analytical modeling and solution of the problem, with the use of continuous approximations. As noted by Daganzo (2004), the main purpose of this type of approach is to obtain reasonable solutions with as little information as possible. Hall (1986) also pointed out that these approximate models are easier for humans to comprehend; we would add that they may provide handy but powerful tools to help solving many complicated decision problems. In particular, in this paper, we will develop relationships to assess the service quality of the two competing operating policies (FRT and DRC) and derive the “critical demand densities”, representing the point where the two services could reasonably be considered equivalent and where a switch from one type of service to the other would be desirable.

2. Literature review

Research on flexible transit services is quite limited and in particular, to our knowledge, there is no research performed on decision methodologies to select the best operating practice for feeders like the DRC. Li and Quadrioglio (2008) approached the problem discussed in this paper by providing a solution based on simulation and sensitivity analyses of a one-vehicle operation. Another work specifically on the DRC, which is the focus of our paper, is Cayford and Yim (2004). These authors surveyed the customers' demand for DRC for the city of Millbrae, California. They also designed and implemented an automated system used for the DRC services. The service uses an automated phone-in-system for reservations, computerized dispatching over a wireless communication channel to the bus driver and an automated callback system for customer notifications. Khattaki and Yim (2004) explored the demand for a consumer oriented personalized DRT (PDRT) service in the San Francisco Bay Area. About 60% of those surveyed were willing to consider PDRT as an option, and about 12% reported that they were “very likely” to use PDRT. Many were willing to pay for the service and highly valued the flexibility in scheduling the service.

Flexible transit services may involve checkpoints. Daganzo (1984) describes a flexible system in which the pick-up and drop-off points are concentrated at centralized locations called checkpoints. The related Mobility Allowance Shuttle Transit (MAST) system allows buses to deviate from the fixed path so that customers within the service area may be picked up or dropped off at their desired locations. According to Koffman (2004), this type of service is also often used and is also known as “Route Deviation”. Quadrioglio et al. (2006) developed bounds on the maximum longitudinal velocity to evaluate the performance and help the design of MAST services by employing continuous approximations. Quadrioglio et al. (2007) developed an insertion heuristic for scheduling MAST services by using control parameters, which properly regulate the consumption of the slack time. Finally, Quadrioglio et al. (2008b) formulated the scheduling of the MAST services as a mixed integer programming with added logic constraints.

Analytical modeling and/or simulation have often been used to analyze flexible transit services. For example, Cortés and Jayakrishnan (2002) proposed and simulated one type of flexible transit called High-Coverage Point-to-Point Transit (HCPPT), which requires the availability of a large number of transit vehicles. Pagès et al. (2006) addressed the real-time mass transport vehicle routing problem and developed a global solution algorithm for the mass transport network design problem. Aldaihani et al. (2004) developed an analytical model that aids decision makers in designing a hybrid grid network

that integrates a flexible demand responsive service with a fixed-route service. Their model is to determine the optimal number of zones in an area, where each zone is served by a number of on-demand vehicles.

Although research on DRC and flexible transit services is quite limited, purely DRT systems have been extensively investigated. Savelsbergh and Sol (1995), Desaulniers et al. (2000), Cordeau and Laporte (2003) provide comprehensive reviews on the proposed methodologies and solutions to deal with these very difficult problems.

In this paper, we utilize continuous approximations as part of our methodology. There is a significant body of work in the literature on continuous approximation models for transportation systems. Most of the work has been developed to provide decision support tools for strategic planning in the design process. Clarens and Hurdle (1975) utilized continuous approximation to design an operating strategy for a commuter bus system. Langevin et al. (1996) provide a detailed overview of the research performed in the field. They concentrate primarily on freight distribution systems, while in this paper we focus on public transport; but most of the issues of interest are common to both fields. Szplett (1984) provides a review of the research performed on continuous models specifically for public transport.

3. System definition

3.1. Service area and demand

The service area is a representation of a residential community modeled as a rectangle of width W and length L located on the side of a main road that is part of a major fixed-route transit service network. The terminal of the feeder transit service, connecting to the major fixed-route transit network, is located in the middle of the left edge of the service area (see Fig. 1). The temporal distribution of the demand is assumed to be a Poisson process with an average rate of λ . We assume that a fraction α of the customers need to be transferred from the service area to the connection terminal (“pick-up” customers) and a fraction $1 - \alpha$ of them in the opposite direction (“drop-off” customers). There are no intrazonal trips, that is, every customer starts or ends the trip at the connection terminal. The customers’ location, either for a pick-up or for a drop-off, has a uniform distribution within the service area. While assuming a temporal Poisson distribution for pick-up customers is very realistic, the drop-off customers would instead reasonably show up in groups according to the arrival of the vehicles serving the outside FRT network. However, with the additional assumption that the number of transit lines passing by the connection terminal is high enough and/or the headways between vehicles are low enough, a Poisson distribution for the arrivals is still a reasonable assumption. The analysis performed in this paper can be updated and refined in future research with a more refined temporal distribution for the customers’ arrivals, but we would not expect a substantial alteration of our results.

3.2. Competing transit policies

We consider two competing operating policies (FRT and DRC) of the transit service. For each one of them we analyze the one-vehicle case and the two-vehicle case. In all our considered scenarios we assume an average speed of the vehicles of v_b . The vehicle dwelling time at each stop is s_f for the FRT and s for the DRC. Dwelling times are defined differently, since we recognize that for the FRT case more passengers would generally be served per stop, therefore we may expect $s \leq s_f$. We also assume that the same type of vehicle(s) is used in all cases.

3.2.1. FRT policy

The FRT operating policy offers continuous service with the vehicle moving back and forth along the route between stop 1 (the connection terminal) and stop N (see Fig. 1). There are $N-2$ stops between 1 and N . The spacing d between adjacent stops

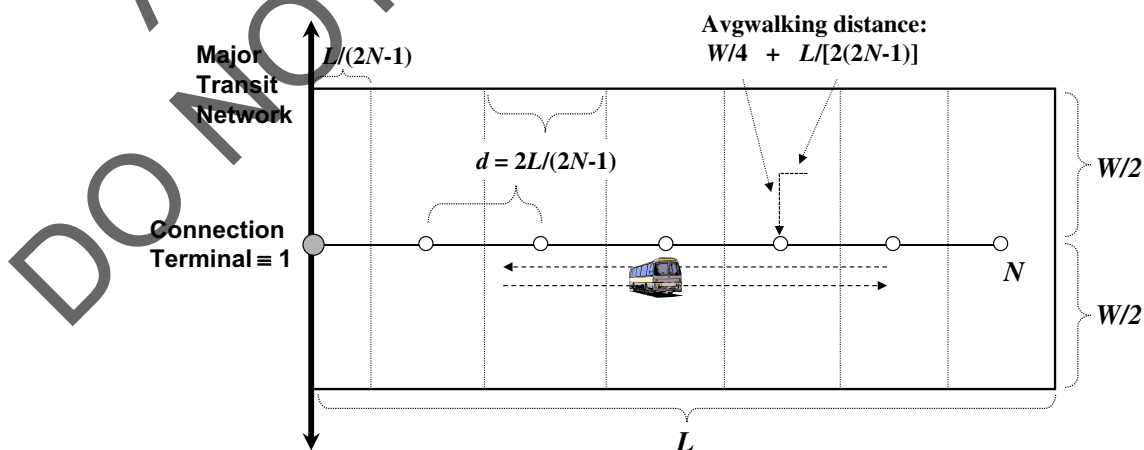


Fig. 1. Service area and FRT service.

is assumed to be constant. N should be determined by the selection of the optimal d , which is often quite hard to derive (see Wirasinghe and Ghoneim, 1981; Kuah and Perl, 1988; Furth and Rahbee, 2000; Saka, 2001). In practice, transit agencies use ranges from 600 ft to 2500 ft in suburban areas (Texas Transportation Institute, 1996). We will derive the optimal d and N by inspection in the results (Section 6) for our considered base case scenarios.

Pick-up customers show up at a random location within the service area and wait there until they need to walk to the nearest station to catch their bus, whose schedule is known to them. Drop-off customers show up and wait at stop 1, ride the bus to the stop nearest to their destination and then walk to their final destination, which is located at random.

In the two-vehicle case we assume that the two buses begin their operations at the same time leaving from stop 1 and stop N , respectively. At any point in time during the operations, the vehicle moving left-to-right performs the drop-off operations (transferring customers from stop 1 to the stops closest to their final destination) and the vehicle moving right-to-left performs the pick-up operations (transferring customers from their stops closest to their origin to stop 1).

3.2.2. DRC policy

The DRC policy provides a shared-ride demand responsive terminal-to-door (and door-to-terminal) service to customers, by picking them up and dropping them off at their desired locations. The vehicle begins and ends each of its trips from the connection terminal. We assume that pick-up customers are able to notify their presence by means of a phone or an internet booking service. Immediately before the beginning of each trip, waiting customers (both pick-up and drop-off ones) are scheduled and the route for the trip in the service area is constructed. There is no planned idle time in between trips. To schedule the requests we assume that the schedule is calculated by an insertion algorithm attempting to minimize the total distance traveled by the vehicle. An insertion heuristic approach is adopted because they are widely used in practice to solve transportation scheduling problems, as they often provide very good solutions compared to optimality, they are computationally fast and they can easily handle complicating constraints (Campbell and Savelsbergh, 2004). Rectilinear movements (as in a Manhattan network) are assumed and often chosen instead of Euclidean ones, since they better estimate distances traveled in real road networks and generally provide good approximations (see Quadrifoglio et al., 2008a).

For the two-vehicle case, we divide the service area into two zones with width W and length $L/2$. Zone 1 is adjacent to the connection terminal and Zone 2 on the right of it (see Fig. 2). Each vehicle serves a zone and operations are scheduled with an insertion heuristic algorithm like the one-vehicle case. Vehicles operate continuously and alternate their operations among zones. This means that each vehicle's cycle would start from stop 1, schedule its service for Zone 1, serve Zone 1 (while the other vehicle is serving Zone 2), come back to stop 1, board waiting drop-off customers for Zone 2, move to Zone 2, schedule its service for Zone 2, serve Zone 2 (while the other vehicle is serving Zone 1) and come back to stop 1.

3.3. Performance measures

The performance of a transit system can roughly be considered as a combination of operating costs and service quality. The relative weight assigned to each of those two categories is a disputed matter and can differ between public transportation agencies and privately owned ones. However, in this paper, we may assume the operating costs to be equivalent for the two competing transit services. The assumption is reasonable in our comparisons, because the vehicle is assumed to be the same and run continuously during the operations for both service policies at the same average speed v_b and the demand served is also the same. We recognize that the FRT will have a shorter cycle and therefore may need a slightly smaller vehicle for its operations. The bus stop infrastructure for the FRT may also bring additional cost, but it would be a small portion in the long term service operation. Thus, other than possible negligible differences, we do not see a major disparity of the operating costs between the two cases which would cause our assumption to be unreasonable.

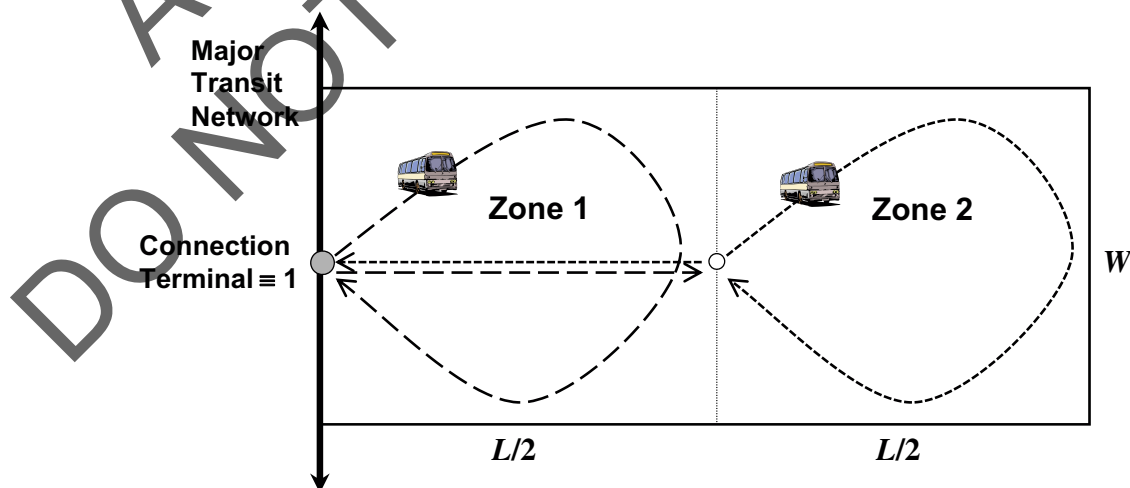


Fig. 2. DRC policy for the two-vehicle case.

Therefore, the comparison between the two services can be made by considering only the service quality provided to customers. If we disregard other possible sources of noise that could influence customers' perceptions and opinions, the service quality can be expressed as a combination of the following performance measures:

- $E(T_{wk})$: expected value of walking time of the passengers to/from their closest bus stop from/to their destination.
- $E(T_{wt})$: expected value of waiting time of the passengers from their ready time to their pick-up time (subtracting the walking time).
- $E(T_{rd})$: expected value of ride time of the passengers from pick-up to drop-off.

Generally, needed transfers between vehicles to complete a trip are a major service quality factor as well, but there are none in this case. Thus, the service quality provided to customers is represented by the utility function U defined as the weighed sum of the above terms:

$$U = w_{wk} \times E(T_{wk}) + w_{wt} \times E(T_{wt}) + w_{rd} \times E(T_{rd}). \quad (1)$$

Lower values of U indicate a better level of service. The assessments of the weights (w_{wk} , w_{wt} and w_{rd}) are generally difficult to make, as they are dependent upon several factors, they are not unique for all cases and they can change dynamically depending on the circumstances. For example: the walking time could be considered more or less acceptable (thus, with a different relative weight), depending on the safety or the weather conditions of a certain area and/or the profile of the customers. However, the weight assignment is not the scope of this paper. We wish to provide decision makers with tools which will help them decide the proper service policy, once they have selected the proper weights for their scenario. A more detailed discussion for the weights can be found in two recent studies, [Wardman \(2004\)](#) and [Guo and Wilson \(2004\)](#).

In the next sections we will focus on the analytical computation of U for both competing policies, so we can make a comparison.

4. Analytical modeling for the one-vehicle case

4.1. FRT

In this section we calculate the expected values of the three performance measures $E(T_{wk})$, $E(T_{wt})$, $E(T_{rd})$ for the one-vehicle case when a FRT operating policy is adopted.

Spacing between adjacent stops is given by (see [Fig. 1](#))

$$d = \frac{2L}{2N - 1}. \quad (2)$$

Assuming that customers would walk to the nearest bus stop with a rectilinear path, the expected value of the walking time $E(T_{wk})$ is

$$E(T_{wk}) = \frac{1}{2v_{wk}} \left(\frac{L}{2N - 1} + \frac{W}{2} \right), \quad (3)$$

where v_{wk} is the average walking speed.

Given that the bus dwelling time at each station is s_f , the cycle time of the journey beginning and ending at stop 1 is

$$C = 2(N - 1) \left[\frac{2L}{v_b(2N - 1)} + s_f \right], \quad (4)$$

since, for each cycle, there are $2(N - 1)$ stops to be visited and $2(N - 1)$ segments of distance d , given by (2), to be traveled.

Since customers walk to the nearest FRT bus stop, those residing in the portion of the service area closest to stop 1 (the half-zone whose size is $1/(2N - 1)$ in proportion to the total service area, shown in [Fig. 1](#)) would walk directly to stop 1. Therefore, the waiting and riding time for these customers are zero. The expected value of the waiting time is $C/2$ for other customers. Thus, the expected value of the waiting time for pick-up customers, drop-off customers and all customers (subscripts p and d denote pick-up and drop-off customers respectively; subscript wt denotes waiting time) are

$$E(T_{wt}^p) = E(T_{wt}^d) = \left(1 - \frac{1}{2N - 1} \right) \frac{C}{2}, \quad (5)$$

$$E(T_{wt}) = \alpha E(T_{wt}^p) + (1 - \alpha) E(T_{wt}^d) = \frac{2(N - 1)^2}{2N - 1} \left[\frac{2L}{v_b(2N - 1)} + s_f \right]. \quad (6)$$

Customers residing in the half-zone closer to stop 1 would ride zero time. Riding time between two consecutive stops is $C/[2(N - 1)]$; so, customers residing in a zone closer to stop $i = 2$ to N , would ride $(i - 1) * C/[2(N - 1)]$ time. Applying conditional probability, the expected value of the ride time for pick-up customers, drop-off customers and all customers (subscript rd denotes riding time) are

$$E(T_{rd}^p) = E(T_{rd}^d) = \frac{1}{2N-1} \times 0 + \frac{2}{2N-1} \sum_{i=2}^N \left[\frac{(i-1)C}{2(N-1)} \right] = \frac{NC}{2(2N-1)}, \quad (7)$$

$$E(T_{rd}) = \alpha E(T_{rd}^p) + (1-\alpha)E(T_{rd}^d) = \frac{N(N-1)}{2N-1} \left[\frac{2L}{v_b(2N-1)} + s_f \right]. \quad (8)$$

4.2. DRC

The calculation of the expected values of the performance measures for the demand responsive operating policy is not straightforward, due to the fact that at each cycle, the vehicle performs a different tour, to serve the demand uniformly but randomly distributed across the service area. However, it is possible to provide good estimates by following a methodology similar to the one adopted in Quadrifoglio et al. (2006). In this paper, authors proved that the distance traveled by a vehicle traveling along a corridor to serve uniformly distributed demand scheduled with an insertion heuristic algorithm (attempting to minimize the total distance traveled) is upper bounded and closely approximated (especially for lower densities) by the distance traveled by the vehicle following a rectilinear “no-backtracking policy”, which forbids backwards movements with respect to the current forward direction and, therefore, serves the customers in order of their horizontal coordinate. In our case, since the vehicle is performing a cycle beginning and ending at stop 1, following the guidelines suggested in Daganzo (2004), we assume that the vehicle would move through the upper half of the region in a no-backtracking policy left-to-right, and move through the bottom half in a no-backtracking policy right-to-left.

Let n be the number of customers served per cycle by the DRC vehicle. Since their spatial distribution is assumed to be uniform, if x_i is the horizontal coordinate within the service area ($0 \leq x_i \leq L$) of customer i (with $i = 1, \dots, n$), the expected value of the maximum horizontal distance that the vehicle will need to travel can be derived as follows:

$$\begin{aligned} E[\max(x_i)|i = 1, \dots, n] &= \int_0^L \{P[\max(x_i)|i = 1, \dots, n] \geq t\} dt = \int_0^L \{1 - P[\max(x_i)|i = 1, \dots, n] \leq t\} dt \\ &= \int_0^L \left\{ 1 - \prod_{i=1}^n [P(x_i) \leq t] \right\} dt = \int_0^L \left(1 - \left(\frac{t}{L}\right)^n \right) dt = L \frac{n}{n+1}. \end{aligned} \quad (9)$$

Customers are uniformly distributed within the whole service area. Let y be the random variable indicating the vertical distance between any pair of customers within the upper or lower half of the service area. We have that $E(y) = W/6$. Let y' indicate the vertical distance between stop 1 (located at $W/2$) and the first or last customer in the schedule. We have that $E(y') = W/4$. Finally, let y'' indicate the vertical distance between the last customer served in the upper half and the first customer served in the lower half. We have that $E(y'') = W/2$. See Fig. 3.

If D represents the expected total rectilinear distance per cycle for a no-backtracking policy, C is the expected cycle time and λ is the average customer demand rate, the following relationships hold:

$$D = 2L \frac{n}{n+1} + 2 \frac{W}{4} + \frac{W}{2} + (n-2) \frac{W}{6} = 2L \frac{n}{n+1} + \frac{2W}{3} + \frac{W}{6} n, \quad (10)$$

$$C = \frac{D}{v_b} + (n+1)s, \quad (11)$$

$$n = \lambda C. \quad (12)$$

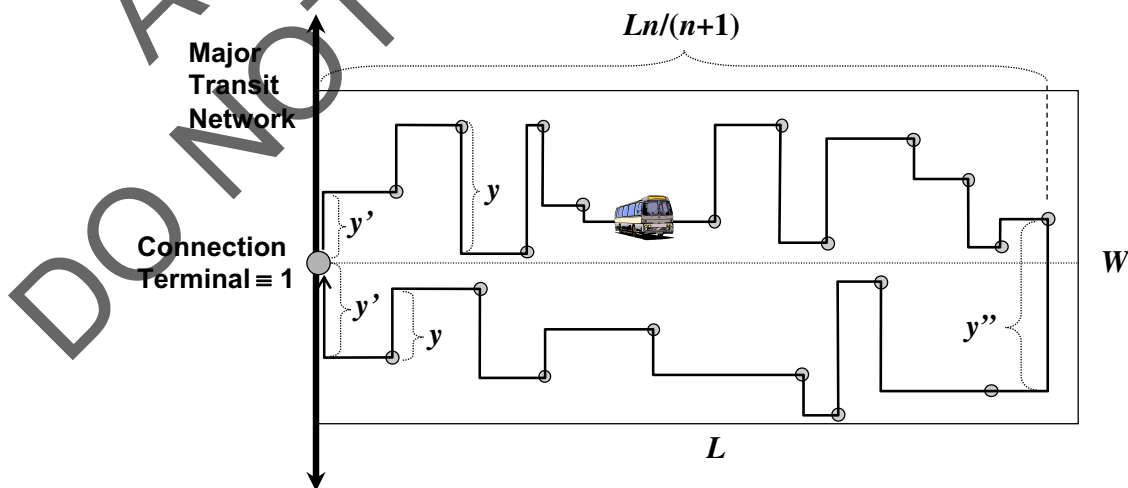


Fig. 3. No-backtracking policy.

Pick-up customers need to wait $E(T_{wt}^p) = C/2 + C/2003DC$, since they will wait an average of $C/2$ from their show-up time to the end of the previous cycle and an additional average of $C/2$, waiting for the vehicle to reach them. We recognize that a small portion of them may be inserted in the schedule of the vehicle already en route, should a real time information system and the current operating conditions allow that; however, we assume that pick-up customers are scheduled in the cycle beginning after their show-up time. In this way the estimated $E(T_{wt}^p)$ represents an approximation and an upper bound of the actual one. Drop-off customers will instead need to wait an average of $E(T_{wt}^d) = C/2$, since they will show up and wait at the connection stop 1 uniformly from time 0 to C of the previous cycle.

Pick-up customers will ride an average of $E(T_{rd}^p) = C/2$, since they can be picked up uniformly anytime from time 0 to C of their cycle. Similarly, drop-off customers will ride an average of $E(T_{rd}^d) = C/2$.

Thus, the expected values of the total waiting time and riding time are

$$E(T_{wt}) = \alpha E(T_{wt}^p) + (1 - \alpha)E(T_{wt}^d) = (1 + \alpha)\frac{C}{2}, \tag{13}$$

$$E(T_{rd}) = \alpha E(T_{rd}^p) + (1 - \alpha)E(T_{rd}^d) = \frac{C}{2}. \tag{14}$$

In order to derive C , we need to solve the system of equations composed by (10)–(12). In doing so we obtain a quadratic equation in the form of

$$aC^2 + bC + c = 0, \tag{15}$$

where

$$a = \lambda \left[\lambda \left(\frac{W}{6} + sv_b \right) - v_b \right], \tag{16}$$

$$b = \lambda \left(\frac{5W}{6} + 2L + 2sv_b \right) - v_b, \tag{17}$$

$$c = \frac{2W}{3} + sv_b. \tag{18}$$

Two obvious conditions should be satisfied: $C > 0$ and $b^2 - 4ac \geq 0$. However, a closed-form expression for C is not easy to derive. Here below we provide two approximations which enable us to derive it.

Approximation 1. In Eq. (10) we could reasonably assume that

$$\frac{n}{n+1} \cong 1, \tag{19}$$

approximating (overestimating) D by an error factor of $2L/(n+1)$, which becomes increasingly negligible with increasing n and becomes zero for $n \rightarrow \infty$. The approximate cycle time \tilde{C} so obtained would be an upper bound of the actual cycle time C and thus still an upper bound of the actual cycle time obtainable by an insertion heuristic. After rearranging (10) with the above approximation (19) and combining it with (11) and (12), we are able to obtain a closed-form expression for the approximate cycle time:

$$\tilde{C} = \frac{2W/3 + 2L}{v_b - \lambda(W/6 + sv_b)} \frac{1}{\lambda[W/(6sv_b) + 1] - 1/s} = \frac{sv_b + 2W/3 + 2L}{v_b - \lambda(W/6 + sv_b)}. \tag{20}$$

Approximation 2. Still applying (19), we substitute $2W/3$ with $(2W/3)n/(n+1)$ in (10), approximating (underestimating) D by an error factor of $(2W/3)/(n+1)$, and $(n+1)s$ with ns in Eq. (11), approximating (underestimating) C by a factor of s . Then, we obtain another closed-form expression for the approximate cycle time:

$$\tilde{C} = \frac{2W/3 + 2L}{v_b - \lambda(W/6 + sv_b)} \frac{1}{\lambda}. \tag{21}$$

Fig. 4 shows how well the closed-form values obtained by “Approx1” (20) and “Approx2” (21) approximate the “Rigorous C ” obtained by numerical methods. Increasing n reduces the error. Since the relationship between the parameters of the error factors introduced by the two approximations is, generally, $2W/3 \ll 2L$ and s is also small, then \tilde{C} given by (21) is closer to C .

The approximate values $E(\tilde{T}_{wt})$ and $E(\tilde{T}_{rd})$ can be obtained by substituting C with \tilde{C} in (13) and (14).

$E(T_{wk})$ and $E(T_{wk})$ are zero, since the DRC policy offers a terminal-to-door service and no walking is necessary.

4.3. Critical demand

We obtain the utility function for the FRT policy by substituting (3), (6), and (8) in (1); similarly, we obtain the utility function for the DRC policy by substituting (13) and (14) in (1). We can now equate these two expressions and solve for λ . The obtained value λ_c represents the critical demand rate at which the two policies would be equivalent in terms of service quality provided to customers.

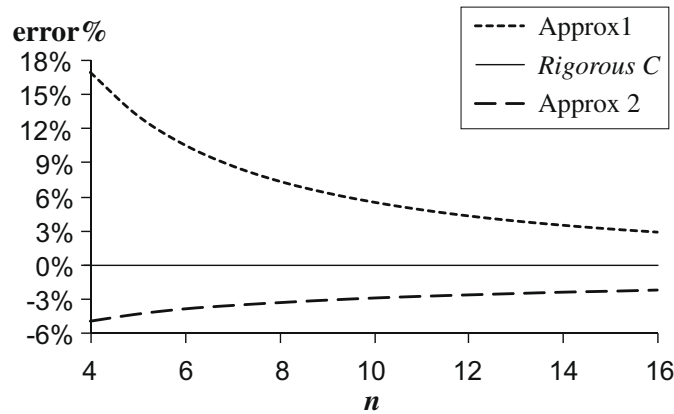


Fig. 4. Error%: \tilde{C} vs. C.

C does not have a closed-form expression and neither does λ_c , but solutions can be obtained with numerical methods. However, if we use (20) to approximate C, a closed-form expression for the approximation of λ_c can be derived and is

$$\tilde{\lambda}_c = \frac{1}{\frac{W}{6v_b} + S} - \frac{\frac{3v_b s + 2W + 6L}{6v_b s + W} [(1 + \alpha)W_{wt} + W_{rd}]}{\frac{W_{wk}}{2v_b} \left(\frac{L}{2N-1} + \frac{W}{2} \right) + \frac{2(N-1)^2}{2N-1} \left[\frac{2L}{v_b(2N-1)} + S_f \right] \left[\frac{N W_{rd}}{2(N-1)} + W_{wt} \right]} \quad (22)$$

Finally, the critical demand density (cust/hr/mile²) is defined as

$$\rho_c = \frac{\lambda_c}{WL}, \quad (23)$$

and its approximation is $\tilde{\rho}_c = \tilde{\lambda}_c / (WL)$.

ρ_c represents the point at which the two services could reasonably be considered equivalent and where a switch from one type of service to the other would be desirable. For expected demand densities lower than ρ_c , the DRC is the preferred operating policy; otherwise the FRT is the preferred operating policy (a negative value of ρ_c would simply suggest that the FRT policy is always preferred). Typically, demand in residential areas follows a double peak pattern, which would possibly lead to attaining the ρ_c four times a day and suggest four switching times, which can be estimated by comparing actual demand data with (23). In practice, it would be best to keep the times of the switch the same every weekday (maybe different during weekends) to avoid confusing customers. An update every so often (like every semester or year), based on historical demand data, could be appropriate and desirable to maximize the performance of the service.

5. Analytical modeling for the two-vehicle case

5.1. FRT

We assume that the two vehicles have the same average speed v_b , the first vehicle starts from stop 1 and the second vehicle starts from stop N. The cycle time is still represented by (4). The expected values of customer walking time and riding time, $E(T_{wk})$ and $E(T_{rd})$, are the same as the one-vehicle case and represented by (3) and (8). The expected value of the waiting time is

$$E(T_{wt}) = \frac{(N-1)^2}{2N-1} \left[\frac{2L}{v_b(2N-1)} + S_f \right], \quad (24)$$

which is simply half of the one for the one-vehicle case.

5.2. DRC

As for the one-vehicle case, we approximate the insertion heuristic operations with a no-backtracking policy left-to-right on the top half and right-to-left on the bottom half of each zone, as suggested by Daganzo (2004). D and C are the expected distance traveled and expected cycle time for each vehicle to serve both zones of the whole service area. As shown in Fig. 2, vehicles alternate their operations between zones. The following equations are similar to (10)–(12), with $\lambda/2$ instead of λ in (27), since n is defined as the number of customers per cycle per zone, $n/2$ instead of n in (25) and (26), since half of the customers are served by each vehicle in each zone, $2(L/2)$ added in (25), because of the driving needed to switch zone by driving (twice) half the length of the whole service area, and a factor 2 multiplying the square bracket in (25), since D computes the distance traveled for covering both zones:

$$D = 2\frac{L}{2} + 2\left[2\frac{L}{2}\frac{\frac{n}{2}}{\frac{n}{2}+1} + 2\frac{W}{4} + \frac{W}{2} + \left(\frac{n}{2} - 2\right)\frac{W}{6}\right] = 2L\frac{n}{n+2} + \frac{W}{6}n + L + \frac{4W}{3}, \tag{25}$$

$$C = \frac{D}{v_b} + 2\left(\frac{n}{2} + 1\right)s, \tag{26}$$

$$n = \lambda C/2. \tag{27}$$

During a cycle C , both vehicles will visit each zone, so pick-up customers will need to wait an average of $C/4$ from their show-up time to the beginning of the operations of either vehicle within their zone, plus an additional average of $(C - L/v_b)/4$, a fourth of the cycle reduced by L/v_b , which is the total transfer time needed by each vehicle to switch zone, during which there are no pick-up nor drop-off operations. Thus, we have that $E(T_{wt}^p) = C/2 - L/(4v_b)$. Drop-off customers will need to wait an average of $E(T_{wt}^d) = C/4$.

The expected ride time in Zone 1 is $E(T_{rd}^{p-1}) = E(T_{rd}^{d-1}) = (C - L/v_b)/4$, again a fourth of the cycle reduced by the transfer time L/v_b . The expected ride time for customers in Zone 2 is instead $E(T_{rd}^{p-2}) = E(T_{rd}^{d-2}) = (C - L/v_b)/4 + L/(2v_b)$, since they need to spend onboard, in addition, the transfer time $L/(2v_b)$ to ride between the connection terminal and Zone 2. Thus, we have that $E(T_{rd}^p) = E(T_{rd}^d) = [E(T_{rd}^{p-1}) + E(T_{rd}^{p-2})]/2 = C/4$.

Thus, the expected values of the total waiting time and riding time are

$$E(T_{wt}) = \alpha E(T_{wt}^p) + (1 - \alpha)E(T_{wt}^d) = (1 + \alpha)\frac{C}{4} - \alpha\frac{L}{4v_b}, \tag{28}$$

$$E(T_{rd}) = \alpha E(T_{rd}^p) + (1 - \alpha)E(T_{rd}^d) = \frac{C}{4}. \tag{29}$$

Solving the system of equations composed by (25)–(27), we obtain a quadratic equation like (15), where

$$a = \frac{\lambda}{4}\left[\lambda\left(\frac{W}{6} + sv_b\right) - 2v_b\right], \tag{30}$$

$$b = \lambda\left(\frac{5W}{6} + \frac{3}{2}L + 2sv_b\right) - 2v_b, \tag{31}$$

$$c = 2L + \frac{8W}{3} + 4sv_b. \tag{32}$$

A closed-form expression for C is not easy to derive. But, as for Approximation 1 in the one-vehicle case, we can say that

$$\frac{n}{n+2} \cong 1. \tag{33}$$

Rearranging (25) with (33), we are able to obtain a closed-form expression for the approximate cycle time for the two-vehicle case:

$$\tilde{C} = \frac{2sv_b + 4W/3 + 3L}{v_b - (\lambda/2)(W/6 + sv_b)} \tag{34}$$

The approximate values $E(\tilde{T}_{wt})$ and $E(\tilde{T}_{rd})$ can be obtained by substituting C with \tilde{C} in (28) and (29).

5.3. Critical demand

By substituting (3), (24), and (8) in (1) we obtain the utility function for the two-vehicle FRT policy; similarly, by substituting (28) and (29) in (1) we obtain the utility function for the two-vehicle DRC policy. We can now equate the two expressions and solve for λ , to obtain the critical demand rate λ_c for the two-vehicle case. Using (34) to approximate C , a closed-form expression for the approximation of λ_c is

$$\tilde{\lambda}_c = \frac{2}{\frac{W}{6v_b} + s} \frac{\frac{6v_b s + 4W + 9L}{6v_b s + W} [(1 + \alpha)W_{wt} + W_{rd}]}{\frac{W_{wk}}{2v_{wk}} \left(\frac{L}{2N-1} + \frac{W}{2}\right) + \frac{(N-1)^2}{2N-1} \left[\frac{2L}{v_b(2N-1)} + S_f\right] \left(\frac{N W_{rd}}{N-1} + W_{wt}\right) + \frac{\alpha L W_{wt}}{4v_b}}, \tag{35}$$

and ρ_c and $\tilde{\rho}_c$ are derived as in (23).

6. Results

In this section we provide numerical results to validate our analytical modeling (rigorous and approximate) vs. simulation. We performed simulations according to the demand distributions assumed in Section 3.1. χ^2 statistical tests show that the simulated data have the assumed distributions. We performed 30 simulation replications. The resulting 95% confidence half-intervals are about 0.7% of the mean for the simulated utility function values (U). The DRC vehicle serves the demand following a schedule calculated with an insertion heuristic algorithm attempting to minimize the vehicle's total travel distance in each cycle.

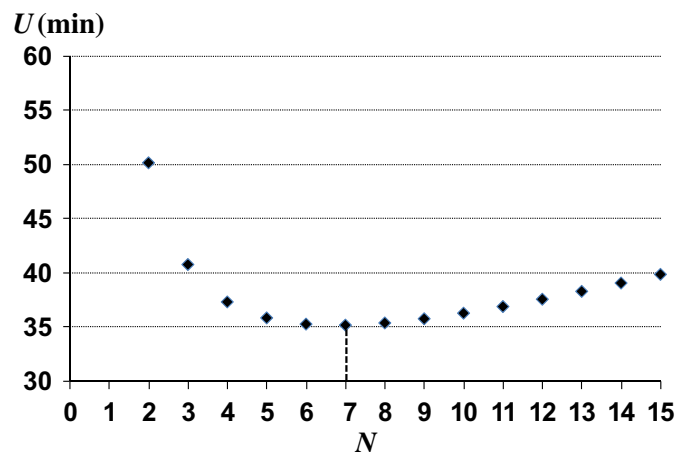


Fig. 5. Optimal N for $L = 2$, $W = 0.5$, $w_{wk} = 3$; one-vehicle case.

6.1. Values of parameters

To represent a residential area, the values of the parameters assumed for our analyses are as follows:

- Pedestrian walking speed $v_{wk} = 2$ miles/h.
- Vehicle speed $v_b = 20$ miles/h.
- Vehicle dwell time at each stop or customer location $s_f = s = 30$ s.
- Service area $L \times W = 1$ mile². We considered $L = 2$ miles and $W = 0.5$ miles, with a ratio $L/W = 4$, as a base case. We also perform sensitivity analyses over L/W .
- We considered a range of different customer demand densities: from 0 up to 90 cust/mile²/h.
- We assume $\alpha = 0.5$, meaning that 50% of the demand are pick-up customers and 50% are drop-off customers (sensitivity results by simulation over the α value can be found in Li and Quadrioglio, 2008).
- We assume $w_{wt} = 1$ and $w_{rd} = 2$. As mentioned, the value of w_{wk} is the most susceptible to variation, due to weather and changing safety conditions; therefore, we consider $w_{wk} = 3$ as a “base case”, but we also perform sensitivity analyses.

6.2. One-vehicle case for $L = 2$, $W = 0.5$

The utility function (1) for the FRT policy is a cumbersome function of the number of stops N , which needs to be minimized to find the optimal spacing d . While we could not obtain a closed-form expression for the optimal N , we plotted U and identified the optimum by inspection; see Fig. 5.

U is a convex function with $U_{\min} = 35.5$ min at $N = 7$, corresponding to $d = 0.31$ miles = 1,637 ft, using (2), which is within the range [600 ÷ 2500 ft] adopted by transit agencies in suburban areas (Texas Transportation Institute, 1996).

We then calculated the utility function curves (1) for the no-backtracking DRC policy for different demand density rates ρ . Four curves were computed: by simulation, by solving (15) by rigorous analytical numerical methods and by solving (15) employing Approximations 1 (20) and 2 (21). Fig. 6 graphically shows the computed utility function curves.

FRT utility function curve (“FRT”) is calculated for $w_{wk} = 3$ (base case) and is flat since it does not depend on the demand. DRC utility function curves (“DRC_Approx1”, “DRC_Rigorous”, “DRC_Simulation”, “DRC_Approx2”) increase with the demand and do not depend on w_{wk} since there is no walking. While we did not assume any capacity constraint in developing our methodology, in all our simulated cases we observed a maximum loading capacity of 25 passengers within our considered range of demand rates. Thus, all our scenarios could have been performed comfortably by a 30-seat bus (for example). Clearly, for higher demand densities, capacity constraints must be taken in consideration, as well as alternative scheduling policies, especially for the DRC.

From the above chart the following observation can be made with regards to the DRC curves.

- The rigorous analytical values are upper bounds for the corresponding simulated values. This is expected, since the no-backtracking policy provides an upper bound of the insertion heuristic algorithm in terms of the distance traveled and consequently in terms of the utility function as well. However, the error is remarkably small (in the range of 1–3%), confirming the good approximations provided by the no-backtracking policy.
- The values of Approximation 1 are an upper bound for the corresponding rigorous values, since our approximate models overestimate the total distance traveled and the gap gets smaller with increasing demand densities, as expected, because of assumption (19).

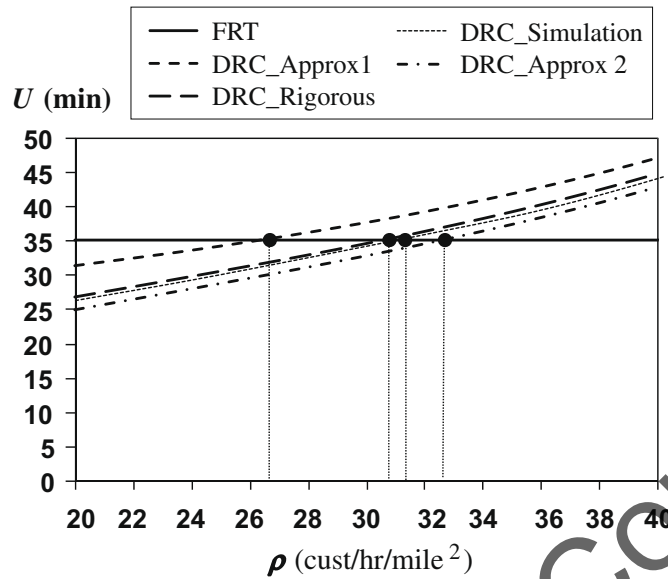


Fig. 6. Utility functions for $L = 2, W = 0.5, w_{wk} = 3$; one-vehicle case.

- The values of Approximation 2 are a lower bound for the corresponding rigorous values, since our approximate models underestimate the total distance traveled and the gap gets smaller with increasing demand densities.
- In general, the four curves are fairly close to each other, which would allow using the developed approximation formulas to estimate the actual utility function values.

The intersection between the DRC curves and the FRT curves represent the critical demand density ρ_c at which the FRT policy and DRC policy have the same utility function values and thus equal performance. For demand densities lower than the critical one, the DRC would be the preferred choice and vice versa. Eq. (23) provides a closed-form expression for these critical demand densities obtained employing Approximation 1 (20). The critical demand densities ρ_c are listed in Table 1, along with the corresponding cycle times C and average number of customers served per cycle n , and shown in Fig. 7, for four different values of w_{wk} (2–5).

The above results show that approximation values $\hat{\rho}_c$ taken from (23) underestimate the rigorous analytical values and the ones derived by simulation. This would mean that the critical “switching point” from DRC to FRT predicted by (23) would be slightly anticipated with increasing demand (and vice versa). Values taken from Approximation 2 would instead do the opposite.

As an illustrative example, consider the scenario where estimated values for the weights are $w_{wk} = 4, w_{wt} = 1, w_{rd} = 2$. The approximate value of the critical demand density given by (23) is 34.3 cust/h/mile². As soon as the demand is expected to drop below this value a switch from a FRT to DRC operating policy would be desirable to maximize the service quality provided to customers. The actual fluctuating daily demand (generally following a double peak pattern) might cross the critical one more than once a day, suggesting several operating switches between policies. While this procedure clearly has intrinsic approximations built in it, it certainly provides a good justifiable estimate. As a validation of our results, for example, the transit operator for a DRC service operated in the city of Winnipeg, Canada, where the service area is close to 1 mile²,

Table 1
 ρ_c, C and n for $L = 2, W = 0.5$; various w_{wk} ; one-vehicle case.

| | Case | $w_{wk} = 2$ | $w_{wk} = 3$ | $w_{wk} = 4$ | $w_{wk} = 5$ |
|--------------------------------------|------------|--------------|--------------|--------------|--------------|
| ρ_c (cust./hr/mi ²) | Simulation | 23.8 | 31.2 | 37.6 | 42.2 |
| | Rigorous | 23.2 | 30.8 | 36.9 | 41.9 |
| | Approx1 | 15.3 | 26.4 | 34.3 | 40.1 |
| | Approx2 | 25.5 | 32.8 | 38.6 | 43.4 |
| C (min) | Simulation | 16.5 | 20.0 | 23.2 | 26.3 |
| | Rigorous | 17.0 | 20.4 | 24.1 | 27.3 |
| | Approx1 | 19.2 | 22.1 | 25.5 | 28.6 |
| | Approx2 | 16.0 | 19.4 | 22.9 | 26.1 |
| n | Simulation | 6.5 | 10.4 | 14.5 | 18.5 |
| | Rigorous | 6.7 | 10.6 | 15.1 | 19.2 |
| | Approx1 | 7.6 | 11.5 | 16.0 | 20.1 |
| | Approx2 | 6.3 | 10.1 | 14.4 | 18.4 |

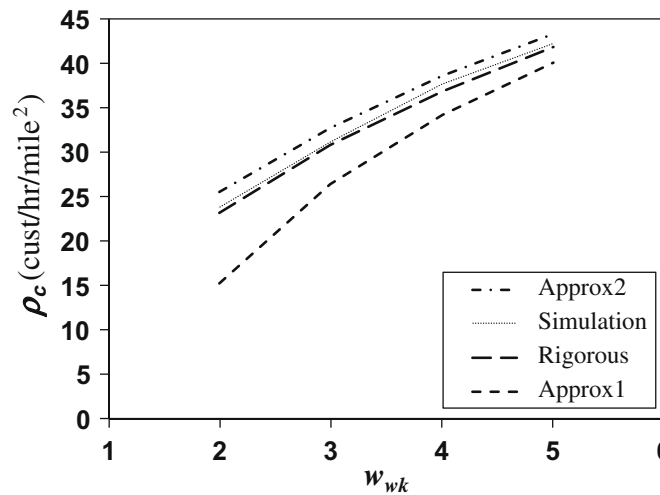


Fig. 7. ρ_c for $L = 2, W = 0.5$; various w_{wk} ; one-vehicle case.

estimated that the DRC policy would be best if operated for up to a maximum of approximately 20 cust/h/mile², which is in the range of our estimates for $w_{wk} = 2$ ($\rho_c = 23.2$ cust/h/mile² by rigorous analytical and $\tilde{\rho}_c = 15.3$ cust/h/mile² by Approximation 1).

6.3. Effect of L/W ratio

In addition to the $L = 2, W = 0.5$ scenario, we produced the critical demand densities, shown in Table 2 and Fig. 8, for $L = 3, W = 0.33$ ($L/W = 9$) and $L = 1, W = 1$ ($L/W = 1$) scenarios to analyze the effects of various L/W ratios. We re-estimated by inspection the optimal N (for the base case $w_{wk} = 3$) in each of the two new cases for the FRT policy, obtaining $N = 5$ (and $d = 0.22$ miles) for $L/W = 1$ and $N = 8$ ($d = 0.4$ miles) for $L/W = 9$.

The critical demand densities decrease with increasing L/W ratio as walking to stations becomes less relevant. For most of the scenarios the rigorous analytical value is very close to the simulated one, strengthening the validity of the no-backtracking policy. We note that for larger L/W ratio and lower w_{wk} value, such as $L/W = 3$ and $w_{wk} = 2$, the Approximation 1 values show significant differences from the ones obtained by simulation, whereas Approximation 2 values show a remarkable match. Therefore, for large L/W ratios, we would recommend adopting the Approximation 2 (or the rigorous analytical values) instead of Approximation 1.

6.4. Two-Vehicle case

We briefly present the results obtained for the two-vehicle case. For the FRT policy the utility function curves (1) were computed with (3), (24), and (8). For $L = 2, W = 0.5$ the optimal $N = 8$ ($d = 0.27$ miles), obtained again by inspection. For the DRC policy the rigorous analytical values of utility function curves (1) were computed with (28) and (29), deriving by numerical methods the cycle times C with (15), (30), (31), and (32); the approximation values to estimate C with \tilde{C} were computed by using (34). As for the one-vehicle case, we also performed simulations to compute the utility function values for two-vehicle DRC policy. Fig. 9 shows the computed utility function curves. The intersection points between the DRC curves and the FRT curve show the critical demand densities which are also listed in Table 3.

As for the one-vehicle case, the approximation values provide an upper bound to the rigorous analytical values. As opposed to the one-vehicle case, the simulated values are slightly larger than the rigorous analytical values; this is caused

Table 2
 ρ_c for various L/W ratios; one-vehicle case.

| L/W | Case | $w_{wk} = 2$ | $w_{wk} = 3$ | $w_{wk} = 4$ | $w_{wk} = 5$ |
|--------|------------|--------------|--------------|--------------|--------------|
| 1/1 | Simulation | 31.2 | 39.1 | 44.7 | 49.2 |
| | Rigorous | 30.6 | 37.1 | 41.1 | 44.4 |
| | Approx1 | 27.8 | 35.8 | 40.6 | 43.8 |
| | Approx2 | 32.7 | 38.9 | 42.2 | 45.4 |
| 3/0.33 | Simulation | 17 | 23.3 | 29.5 | 34.8 |
| | Rigorous | 16.1 | 22.3 | 28.6 | 34.3 |
| | Approx1 | 3.3 | 15.1 | 24.1 | 31.2 |
| | Approx2 | 17.8 | 24.1 | 30.4 | 35.7 |

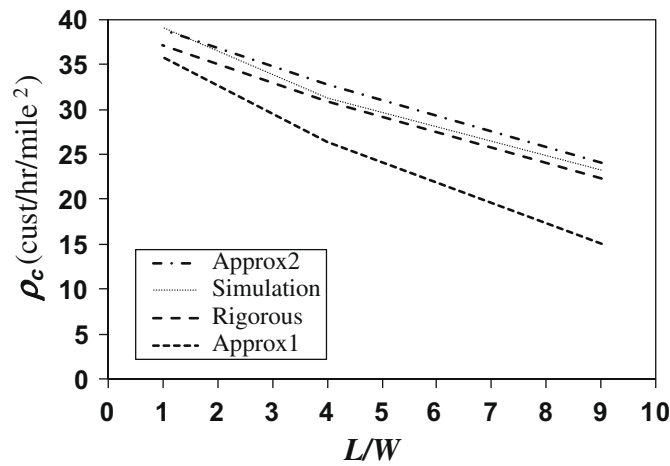


Fig. 8. Effect on ρ_c of various L/W ratios, $w_{wk} = 3$; one-vehicle case.

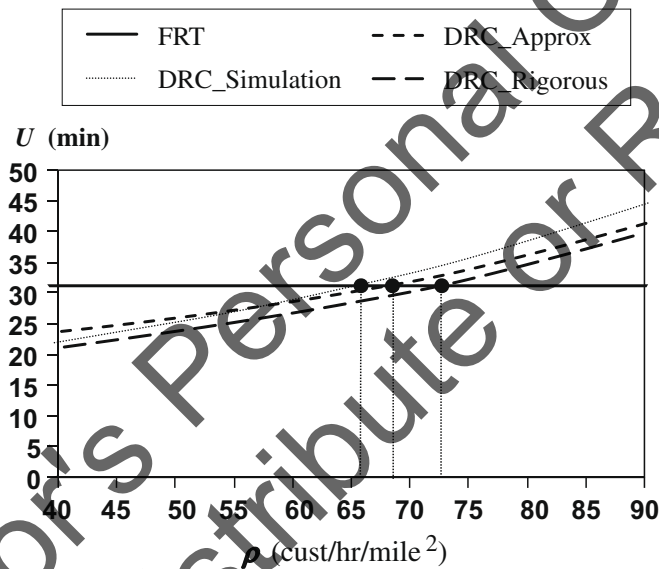


Fig. 9. Utility functions for $L = 2$, $W = 0.5$, $w_{wk} = 3$; two-vehicle case.

Table 3
 ρ_c for $L = 2$, $W = 0.5$; various w_{wk} ; two-vehicle case.

| Case | $w_{wk} = 2$ | $w_{wk} = 3$ | $w_{wk} = 4$ | $w_{wk} = 5$ |
|------------|--------------|--------------|--------------|--------------|
| Simulation | 50.5 | 65.3 | 77.5 | 87.1 |
| Rigorous | 55.7 | 72.5 | 84.3 | 94.2 |
| Approx | 47.8 | 68.1 | 81.5 | 92.2 |

by the existing correlations between the vehicles' operational cycles, which are not captured by our two-vehicle modeling, in which we assumed independency. While the differences are more noticeable than the one-vehicle case, they are still within an acceptable 10% maximum deviation. In addition, the approximation values obtained by (35) are closer to the simulation values than the rigorous ones, which is a good practical result. Finally, by comparing critical demand density's values in Tables 1 and 3, we note that the ρ_c for the two-vehicle case are more than twice as much the ones for the one-vehicle case, suggesting an interesting synergistic effect in favor of the DRC policy.

7. Conclusions

Feeder transit services are generally operated with a demand responsive policy which might be converted to a traditional fixed-route policy for higher demand. In this paper we investigated the conditions that would justify the switch between the

two policies. By employing continuous approximations, we provide an analytical modeling framework of the decision problem to help planners and operators in their choice. We compared the utility functions of the competing operating policies to identify the critical demand densities, representing the switching conditions, that are functions of the parameters of each considered scenario, such as the geometry of the service area, the vehicle speed and also the weights assigned to each term contributing to the utility function: walking time, waiting time and riding time.

Specifically, we derived rigorous analytical values and approximate closed-form expressions of the critical demand density for a range of plausible scenarios. Values obtained for the one-vehicle case with $L = 2$ miles and $W = 0.5$ miles range from 23 to 42 cust/h/mile², slightly underestimate the values obtained by simulation, validating our methodological approach. We also performed sensitivity analyses over different L/W ratios and walking time weights. In addition, our results show a good match with data obtained from a feeder transit service operated in the City of Winnipeg, Canada.

In conclusion, this paper suggests and encourages transit planners and operators to make use of this methodological approach in selecting the correct operating policy for feeders, whose proper design and operations are becoming increasingly important to enhance the performance of the public transportation system network, within modern sprawled urban and suburban areas. The use of our handy but powerful closed-form analytical expression to estimate the critical demand density may not be limited to urban residential areas, but can also be applied to rural regions, with much larger service areas, which traditionally have lower demand rates.

Acknowledgement

The research reported in this paper was supported by the South West University Transportation Center (Texas Transportation Institute) under Grant 473700-00090.

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