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Feeder transit services: Choosing between fixed and demand responsive policy

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ABSTRACT

The Demand Responsive Connector (DRC) connects a residential area to a major transit network through a transfer point and is one of the most often adopted types of flexible transit services. In this paper, analytical and simulation models are developed to assist planners in the decision making process when having to choose between a demand responsive and a fixed-route operating policy and whether and when to switch from one to the other during the day. The best policy is chosen to maximize the service quality, defined as a weighed sum of customer walking time, waiting time and ride time. Based on the results of one-vehicle operations for various scenarios, we have generated critical customer demands, which represent switching points between the competing service policies. Our findings show that the critical demands are in the range from 10 to 50 customers/mile²/h and that a demand responsive policy is more preferred during afternoon peak hours.

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1. Introduction

Most transit systems fall into two broad categories: fixed-route transit (FRT) and demand responsive transit (DRT) systems. Traditional FRT systems are typically more cost efficient because of the predetermined schedule, the large loading capacity of the vehicles and the consolidation of many passenger trips onto a single vehicle (ridesharing). However, the general public considers them to be inconvenient because of their lack of flexibility, since the locations of pick-up and/or drop-off points and/or the service's schedule often do not match the individual rider's desires. DRT systems instead provide desired flexibility with a door-to-door type of service, but they are generally much more costly to deploy. Therefore, they are largely limited to specialized operations such as taxicabs, shuttle vans, dial-a-ride services and paratransit services.

The broad category of "flexible" transit services includes all types of hybrid services that combine pure demand responsive services and fixed-route services. These services have established stop locations and/or established schedules, combined with some degree of demand responsive operation. As opposed to regular fixed-route systems, their use has been quite limited in practice so far.

The Demand Responsive Connector (DRC) is considered a flexible transit service because it operates in a demand responsive fashion within a service area and moves passengers to a transfer point that connects to a fixed-route transit network (see Fig. 1). A survey conducted by Koffman (2004) found that DRC transit service has been operating in quite a few cities and is one of the most commonly adopted types of flexible transit service, especially within low density residential areas, which is the often result of urban sprawl, one of the most evident land-use phenomena occurring in the last few decades. In most cases, the transit line in the service area operates as a FRT service during daytime and switches to a DRC type of service during evenings, nights or early morning, when demand is lower.

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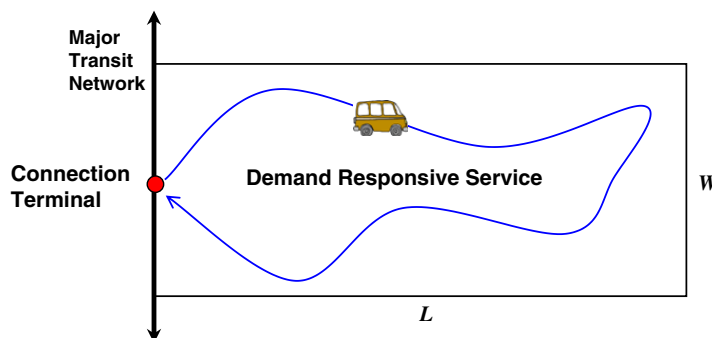


Fig. 1. Demand Responsive Connector (DRC).

In designing such systems, planners often are not sure whether to implement a traditional FRT or a DRC service as a feeder line for the major FRT network and in particular about what conditions would justify a “switch” between FRT and DRC operating modes. In fact, the decision is not straightforward, mainly because the demand for the service is often unknown beforehand and it will depend on the established system itself. In addition, even assuming known demand, it is not clear what would be the best type of service. This is because the balance between operating costs and service quality is frequently hard to evaluate, especially within residential areas with low and sparse demand.

In this paper the authors present a methodology to assist decision makers in choosing among FRT and DRC. Analytical derivations, simulation and sensitivity analyses are developed to assess the service quality of the competing FRT and DRC services for various customer demands and service zone sizes. Then, for each considered scenario, we are able to determine the critical demand densities for which the FRT and DRC services have an equivalent service quality. These critical demand densities represent the point where it would be desirable to switch from one type of service to another and are very useful for planners and decision makers in the selection of the type of feeder service.

2. Literature review

Flexible transit services merge the flexibility of demand responsive transit systems with the low cost operability of fixed-route systems. Koffman (2004) shows that flexible transit services have been used in several cities. We present a review of the work performed on them.

Cayford and Yim (2004) is a specific work on the DRC itself, which is the focus of our paper. These authors surveyed customer demand for DRC for the city of Millbrae, California. They also designed and implemented an automated system used for the DRC services. The service uses an automated phone-in-system for reservations, computerized dispatching over a wireless communication channel to the bus driver and an automated callback system for customer notifications.

Flexible transit services may involve checkpoints. Daganzo (1984) describes a checkpoint demand responsive system where pick-up and drop-off points are at centralized locations called checkpoints. Comparing checkpoint system with fixed-route and door-to-door systems, the author found that fixed-route systems perform best under high demand levels while the door-to-door performs best under low demand levels. A related Mobility Allowance Shuttle Transit (MAST) system has been studied by Quadrioglio (2006, 2007, 2008b). The MAST system allows buses to deviate from the fixed path so that customers within the service area may be picked up or dropped off at their desired locations. According to Koffman (2004), this type of service is often used and is also known as “Route Deviation”. Quadrioglio et al. (2006) developed bounds on the maximum longitudinal velocity to evaluate the performance and help the design of MAST services. Quadrioglio et al. (2007) developed an insertion heuristic for scheduling MAST services by using control parameters, which properly regulate the consumption of the slack time. Finally, Quadrioglio et al. (2008b) formulated the scheduling of the MAST services as a mixed integer programming with added logic constraints. Experiments showed that the developed inequalities achieved 90% reduction of the CPU time for some instances.

Analytical modeling and/or simulation have often been used to analyze flexible transit services. For example, Cortés and Jayakrishnan (2002) proposed and simulated one type of flexible transit called High-Coverage Point-to-Point Transit (HCPPT), which requires the availability of a large number of transit vehicles. Pagès et al. (2006) identified the real-time mass transport vehicle routing problem and developed a global solution algorithm. The mass transport network design problem was formulated and solved by the developed algorithm. Aldaihani et al. (2004) developed an analytical model that aids decision makers in designing a hybrid grid network that integrates a flexible demand responsive service with a fixed-route service. Their model determines the optimal number of zones in an area, where each zone is served by a number of on-demand vehicles.

The DRC has a close relation to DRT systems. Although research on the DRC is quite limited, the DRT systems have been extensively investigated. Savelsbergh and Sol (1995), Desaulniers et al. (2000) and Cordeau and Laporte (2003) provide comprehensive reviews. We summarize a few more papers describing different issues and problem solving approaches to the purely demand responsive services.

Khattak and Yim (2004) explored the demand for a consumer oriented personalized DRT (PDRT) service in the San Francisco Bay Area. About 60% of those surveyed were willing to consider PDRT as an option, about 12% reported that they were “very likely” to use PDRT. Many were willing to pay for the service and highly valued the flexibility in scheduling the service. Sandlin and Anderson (2004) presented a procedure for calculating a serviceability index (SI) for DRT operators based on regional socioeconomic conditions and internal operation data. The SI can be used to evaluate and compare DRT operation. Palmer et al. (2004) studied the DRT system consisting of dial-a-ride programs that transit agencies use for point to point pick-up and drop-off of the elderly and handicapped. Their results of a nationwide survey involving 62 transit agencies show that the use of paratransit computer aided dispatching (CAD) system and agency service delivery provide a productivity benefit. Diana et al. (2006) studied the problem of determining the number of vehicles needed to provide a DRT service with a predetermined quality for the user in terms of waiting time at the stops and maximum allowed detour. Quadrifoglio et al. (2008a) used simulation methods to investigate the effect of using a zoning vs. a no zoning strategy and time window settings on performance measures such as total trip miles, deadhead miles and fleet size. They identified quasi linear relationships between the performance measures and the independent variable, either the time-window size or the zoning policy. Dessouky et al. (2003) demonstrated through simulation that it is possible to reduce environmental impact substantially while increasing operating costs and service delays only slightly for the joint optimization of cost, service, and life cycle environmental consequences in vehicle routing and scheduling of a DRT system.

In this paper we aim to investigate and establish the conditions which would justify the implementation of a demand responsive operating policy for the feeder transit services as opposed to a traditional fixed-route one. To our knowledge, this paper is the first to develop a methodology for solving this problem.

3. System description

3.1. Service area and demand

The service area considered for our study is a representation of a residential community and is modeled as a rectangle of width W and length L (see Fig. 1). The terminal connecting with the outside fixed-route transit network is located at width $W/2$ on the far left of the area. The temporal distribution of the demand is assumed to have a Poisson distribution. We assume that a fraction α of the customers need to be transferred from the service area to the connection terminal (“pick-up” customers) and a fraction $1 - \alpha$ of them will travel in the opposite direction (“drop-off” customers). The customers’ location, either for a pick-up or for a drop-off, has a uniform distribution within the service area. While a Poisson distribution for pick-up customers is very realistic, the drop-off customers would instead reasonably show up in groups according to the arrival of the vehicles serving the outer major FRT network. However, if we assume that the number of lines passing by the connection terminal is high enough and/or the headways between vehicles are low enough then temporal Poisson arrival rate is also a reasonable assumption.

3.2. Competing transit policies

We consider only one vehicle moving at average speed V_{bus} miles/h and stopping at each station for a period of T_s (dwelling time).

3.2.1. FRT policy

The fixed-route transit (FRT) policy offers continuous service with the vehicle moving back and forth along the route between Point A (the connection terminal) and Point B (the last stop at the opposite side of the service area; see Fig. 2). There are a number of stations N (including A and B) and the distance between adjacent stations is a constant (d). The pick-up customers show up at random within the service area and walk to the nearest bus stop. The drop-off customers show up

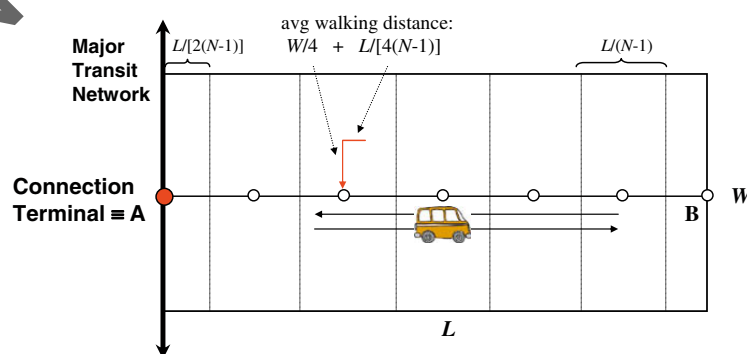


Fig. 2. FRT service.

at terminal A, ride the bus to a stop near their destination and then walk to their final destination, which is located at random within the service area.

3.2.2. DRC policy

The DRC policy provides a demand responsive terminal-to-door (and door-to-terminal) service to customers, by picking them up and dropping them off at their desired locations (see Fig. 1). The vehicle begins and ends each of its trips from the terminal. We assume that pick-up customers are able to notify their presence by means of a phone or internet booking service. Immediately before the beginning of each trip, waiting customers (both pick-up and drop-off ones) are scheduled and the route for the trip in the service area is constructed. There is no planned idle time in between trips. The DRC vehicle keeps running except for the following condition (which rarely happens anyway, generally due to very low demand): if the vehicle returns to the terminal and no customer is requesting a pick-up or a drop-off at any location, then it waits until a customer shows up. To schedule the requests we use an insertion heuristic algorithm described in Section 4.3.

4. Model description

4.1. Measure of performance

If we disregard other possible sources of noise, that could influence customers' perceptions and opinions and we assume the cost of the service is the same for both service policies, the service quality can be considered as a combination of the following measures of performance (MOPs):

- $E(T_{wk})$: expected value of walking time of the passengers needed between their closest bus stop and their destination.
- $E(T_{wt})$: expected value of waiting time of the passengers before pick-up.
- $E(T_{rd})$: expected value of ride time of the passengers from pick-up to drop-off.

Generally, needed transfers between vehicles to complete a trip are a major service quality factor as well, but there are none in this case. Thus, the service quality provided to customers is represented by the utility function U defined as the combination of these MOPs with weights w_1 , w_2 , and w_3 :

$$U = w_1 \times E(T_{wk}) + w_2 \times E(T_{wt}) + w_3 \times E(T_{rd}). \quad (1)$$

Weight assessments are generally difficult to perform, because they are dependent upon several factors and they are not unique for all cases. For example: the walking time could be considered more or less acceptable (thus, with a different relative weight), depending on the safety or the weather conditions of a certain area and/or the profile of the customers. However, the weight assignment is not the scope of this paper. We wish to provide decision makers with tools which will help them decide the proper service policy, once the proper weights for the chosen area have been selected depending on the circumstances.

It is possible to compare the competing transit services by analytical modeling and simulation analysis for different values of the variables and parameters defined above. The same set of customers can be assumed to be served by the FRT or the DRC policy and we can calculate the utility function U for each case. A lower U value would indicate the better transit service policy.

4.2. FRT analytical modeling

We derived analytical relationships of the expected value of $E(T_{wk})$, $E(T_{wt})$ and $E(T_{rd})$ for the FRT policy as follows. The values obtained by the following formulas have been fully matched by simulation results.

4.2.1. Customer walking time

As illustrated by Fig. 2, for a uniform spatial distribution, the expected value of the customer walking time is

$$E(T_{wk}) = \frac{1}{V_{wk}} \left(\frac{L}{4(N-1)} + \frac{W}{4} \right), \quad (2)$$

where V_{wk} is the customer walking speed; N is number of FRT bus stations assuming equal space between bus stations. The derivation of Eq. (2) is a simple geometrical calculation.

Since the bus dwelling time at each station is T_s , the cycle time of the journey beginning at terminal A and back is

$$T_c = \frac{2L}{V_{bus}} + 2(N-1)T_s. \quad (3)$$

The derivation of the expected values for the waiting time and riding time depends upon the relationship between the values assumed for the weights w_2 and w_3 . As previously mentioned, our scope is not to assess the weights, but to provide analytical tools given their assumed values.

4.2.2. $w_2 < w_3$ case

This case would mean that customers prefer to spend their time waiting rather than being on the vehicle. This is a reasonable assumption if the waiting location is a comfortable one, like at home or at nicely built connection terminal. The expected value of the ride time for pick-up customers, drop-off customers and all customers are:

$$E(T_{rd-1}^p) = \frac{T_c}{4}, \tag{4}$$

$$E(T_{rd-1}^d) = \frac{T_c}{4}, \tag{5}$$

$$E(T_{rd-1}) = \alpha E(T_{rd-1}^p) + (1 - \alpha) E(T_{rd-1}^d) = \frac{1}{2} \left[\frac{L}{v_{bus}} + (N - 1) T_s \right]. \tag{6}$$

The expected value of the waiting time for pick-up customers, drop-off customers and all customers are instead:

$$E(T_{wt-1}^p) = \left[\frac{1}{2} - \frac{1}{4(N - 1)} \right] T_c, \tag{7}$$

$$E(T_{wt-1}^d) = \left[\frac{1}{2} - \frac{1}{4(N - 1)} \right] T_c, \tag{8}$$

$$E(T_{wt-1}) = \alpha E(T_{wt-1}^p) + (1 - \alpha) E(T_{wt-1}^d) = \left[1 - \frac{1}{2(N - 1)} \right] \left[\frac{L}{v_{bus}} + (N - 1) T_s \right]. \tag{9}$$

4.2.3. $w_2 > w_3$ case

This case would instead mean that customers would spend their time onboard rather than waiting. This could be the case when most of the waiting occurs at possibly unsafe locations, maybe at night and/or with adverse weather conditions. Eqs. (4)–(9) are then recalculated by employing conditional probability. For pick-up customers, we skip for brevity the mathematical passages (see more details in the Appendix A) and only provide the resulting relationship which is

$$E(T_{rd}^p) = \left[\frac{5}{12} - \frac{1}{6(N - 1)^2} \right] T_c. \tag{4a}$$

Similarly, for drop-off customers, the expected value of ride time is

$$E(T_{rd}^d) = \frac{1}{4} T_c. \tag{5a}$$

Therefore, the expected value of customer ride time is

$$E(T_{rd}) = \alpha E(T_{rd}^p) + (1 - \alpha) E(T_{rd}^d) = \left[\frac{\alpha}{3} \left(1 - \frac{1}{(N - 1)^2} \right) + \frac{1}{2} \right] \left[\frac{L}{v_{bus}} + (N - 1) T_s \right]. \tag{6a}$$

In an analogous fashion, it is possible to derive the expected value of the waiting time for pick-up customers, drop-off customers and all customers:

$$E(T_{wt-2}^p) = \left[\frac{1}{3} - \frac{1}{4(N - 1)} + \frac{1}{6(N - 1)^2} \right] T_c, \tag{7a}$$

$$E(T_{wt-2}^d) = E(T_{wt-1}^d) = \left[\frac{1}{2} - \frac{1}{4(N - 1)} \right] T_c, \tag{8a}$$

$$E(T_{wt-2}) = \alpha E(T_{wt-2}^p) + (1 - \alpha) E(T_{wt-2}^d) = \left[\frac{\alpha}{3} \left(\frac{1}{(N - 1)^2} - 1 \right) - \frac{1}{2(N - 1)} + 1 \right] \left[\frac{L}{v_b} + (N - 1) T_s \right] \tag{9a}$$

4.3. DRC simulation modeling

The analytical derivation of the terms of the utility function for the DRC performance is very difficult because of the embedded vehicle routing problem. Therefore, we use simulation to replicate the operations of the DRC vehicle to pick up or drop off customers and derive the MOPs.

The simulation is developed with MATLAB (The MathWorks, Inc., 2007) software. We performed 30 replications of 100 DRC cycles, scheduling 2000 customers each. While the actual operations will not last so long, the long simulation time is needed to generate stable values of the means of the output parameters.

The DRC simulation model embeds the following insertion heuristic algorithm to schedule the requests of customers. Rectilinear distances are used because they are more similar to the road network than the Euclidean distances are. In fact,

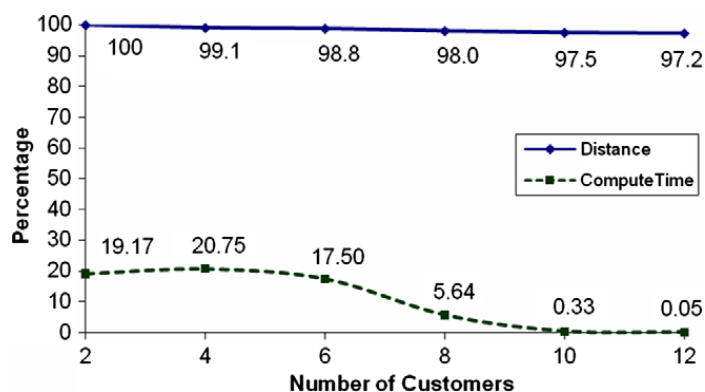


Fig. 3. Performance comparison of insertion algorithm vs. optimality.

as shown in Quadrioglio et al. (2008a), a rectilinear movement of the vehicle is a good approximation of the reality. We assume no “real time” scheduling; customers showing up while the DRC vehicle is performing a trip are scheduled and served in the following trip. However, this option can be considered at a later stage for a further improvement of the operations.

Insertion algorithm:

Let $C_1, C_2, C_3, \dots, C_n$, denote n customers and $T_1 < T_2 < T_3 < \dots < T_n$, their show-up times. The insertion algorithm creates the customer sequence choosing the minimum additional distance at each insertion step in an $O(n^2)$ fashion, as follows:

- (1) Insert C_1 : AC_1A is the only possible route.
- (2) Insert C_2 : Possible routes include AC_2C_1A , and AC_1C_2A . Find the route R_2 with the minimum DRC running distance among the two possible routes. Suppose R_2 is Route AC_1C_2A .
- (3) Insert C_3 : Possible routes include $AC_3C_1C_2A$, $AC_1C_3C_2A$, and $AC_1C_2C_3A$. Find the route R_3 with the minimum DRC running distance among the three possible routes.
- (4) ...
- (n) Insert C_n : Suppose the route R_{n-1} is generated by inserting C_{n-1} ; Insert C_n to the route R_{n-1} . Find the route R_n with the minimum DRC running distance among the n possible routes.

The algorithm complexity is polynomial $O(n^2)$, which can be solved almost instantaneously by any modern PC. An insertion heuristic approach is used because they are widely used in practice to solve scheduling problems, as they often provide very good solutions compared to optimality, they are computationally fast and they can easily handle complicating constraints (Campbell and Savelsbergh, 2004). To verify that in our case, we provide an assessment of our heuristic, which is done by comparing its performance against optimality, obtained by solving the related vehicle routing problem (VRP), notoriously a NP-hard problem, using CPLEX 11.2, a state of the art optimization software, on the same PC.

Fig. 3 shows the comparisons of the computation time and the achieved minimum distance, which are generated by the insertion algorithm and the CPLEX solutions. When the number of customers for a cycle increases from 2 to 12, the minimum distance obtained by the insertion algorithm decreases from 100% to 97.2% of that obtained by the CPLEX. When the number of customers is in the range of 2–6, the computation time of the insertion algorithm is close to 20% of that of the CPLEX. However, for 12 customers, the CPLEX computation time is about 107 s (for a computer with Pentium® 4 2.80 GHz CPU and 1.00 GB RAM), and is 2000 times longer than the computation time of the insertion algorithm. If there are more than 12 customers, the CPLEX computation time is too long for practical use. Therefore, the insertion algorithm is used, because of its very fast computation time and its close-to-optimality performance in our demand range of interest.

5. Result analysis

5.1. Parameter values

The default values used for analysis are: FRT bus station distance ($d = 0.25$ miles), pedestrian walking speed ($V_{wk} = 2$ miles/h), bus running speed ($V_{bus} = 20$ miles/h), bus dwell time at each station or customer location ($T_s = 30$ s) and the result analysis is based on the following assumed values of the other parameters, which can be of course modified to suit any other particular scenario:

- (1) The service area is 1 mile². However, we considered three different W/L ratios: with the length L equal to 1, 2, 4 miles and the width W to 1, 0.5, 0.25 miles, respectively.

- (2) Approximating the results on the basis of two recent studies, Wardman (2004) and Guo and Wilson (2004), we assume $w_2 = 1$ and $w_3 = 2$ and therefore use the formulas in Section 4.2.2. As mentioned, the value of w_1 is the most susceptible to variation, due to weather and changing safety conditions; therefore, we consider $w_1 = 3$ as a “base case”, but we also perform sensitivity analyses.
- (3) We consider $\alpha = 0.5$ as a “base case”, meaning that 50% of the demand are pick-up customers and 50% are drop-off customers. The number of pick-up customers may be not equal to the number of drop-off customers, such as in morning or afternoon peak hours. We investigate the effects of various α values.

5.2. Analytical results

With Eqs. (2), (6), and (9), the FRT performances are calculated. The results with $\alpha = 0.5$ are listed in Table 1.

5.3. Simulation results

For the designed scenarios, the simulations generated performances for both FRT and DRC. With $\alpha = 0.5$, $E(T_{wk})$, $E(T_{wt})$ and $E(T_{rd})$ are listed in Tables 2–4. $E(T_{wk})$ for DRC is zero, since it serves customers at their desired locations.

From Tables 1–4, we can make the following observations:

- (1) By comparing Table 1 with Tables 2–4, it is possible to verify the validity of the analytical values obtained by Eqs. (2)–(9) with the simulation results for FRT.
- (2) The MOPs of FRT are independent of the demand. This is expected because the bus capacity is assumed to be large enough not to be a binding constraint. We verified that the maximum passenger load in any segment, considering all the performed simulations, does not exceed the value 25. However, in most cases much smaller capacity vehicles are needed to serve all the demand. For brevity, we are not providing the maximum required capacity for each case, since this is not the primary scope of this paper.
- (3) For FRT, $E(T_{wk})$ decreases with the decrease of service area width W . This is also expected, since narrower service areas would result in shorter walking distances to the closest stop.

Table 1
Analytical results of FRT performance.

MOP (min)	$L = 1$ mile $W = 1$ mile	$L = 2$ miles $W = 0.5$ miles	$L = 4$ miles $W = 0.25$ miles
$E(T_{wk})$	9.375	5.625	3.75
$E(T_{wt})$	4.375	9.375	19.375
$E(T_{rd})$	2.5	5.0	10.0

Table 2
Simulation results for $L = 1$ mile and $W = 1$ mile ($\alpha = 0.5$).

MOP (min)		Demand (customer/mile ² /h)			
		24	32	40	48
FRT	$E(T_{wk})$	9.38	9.39	9.37	9.39
	$E(T_{wt})$	4.39	4.38	4.37	4.37
	$E(T_{rd})$	2.50	2.50	2.50	2.50
DRC	$E(T_{wk})$	9.02	12.12	16.28	22.44
	$E(T_{rd})$	6.07	8.11	10.82	14.93

Table 3
Simulation results for $L = 2$ miles and $W = 0.5$ miles ($\alpha = 0.5$).

MOP (min)		Demand (customer/mile ² /h)					
		24	28	32	36	40	42
FRT	$E(T_{wk})$	5.63	5.63	5.63	5.62	5.62	5.61
	$E(T_{wt})$	9.39	9.37	9.38	9.40	9.35	9.38
	$E(T_{rd})$	4.99	5.00	5.01	5.01	5.00	5.00
DRC	$E(T_{wt})$	12.52	13.95	15.43	17.00	18.98	19.92
	$E(T_{rd})$	8.40	9.34	10.29	11.30	12.63	13.30

Table 4
Simulation results for $L = 4$ miles and $W = 0.25$ miles ($\alpha = 0.5$).

MOP (min)		Demand (customer/mile ² /h)						
		16	18	20	22	24	26	30
FRT	$E(T_{wk})$	3.75	3.75	3.74	3.74	3.75	3.75	3.75
	$E(T_{wt})$	19.32	19.37	19.37	19.36	19.41	19.37	19.39
	$E(T_{rd})$	9.99	10.01	10.03	9.99	10.01	10.02	10.02
DRC	$E(T_{wt})$	19.41	20.39	21.14	21.97	22.85	23.81	25.48
	$E(T_{rd})$	13.01	13.62	14.17	14.67	15.28	15.84	17.02

- (4) For FRT, $E(T_{wt})$ and $E(T_{rd})$ increase with the increase of service area length L , because the FRT cycle time increases.
- (5) For DRC, $E(T_{wt})$ and $E(T_{rd})$ increase with the increase of customer demand, since the DRC trip time is proportional to the number of customers served each time.
- (6) For DRC, $E(T_{wt})$ and $E(T_{rd})$ increase with the increase of L , since a narrower area is less compact, leading to longer trips.

5.4. Critical customer demand density

Combining the MOPs with the assumed weights, it is possible to calculate the utility function U for both services and identify the best type of service for each scenario. While U for FRT is independent of the demand, the U for DRC is a monotonic increasing function of the demand. It has already been established that lower demand densities are more suitable for DRC than FRT types of services, but the identification of a “switching point” between the two services is not always so clear. Thus, we label the customer demand as “critical” when the FRT and DRC have the same utility function value. For demands lower than the critical demand, the DRC service is better than the FRT service. The FRT service is better than the DRC service for demands greater than the critical demand.

Figs. 4–6 show the utility function values of FRT and DRC for various demands and w_1 with $\alpha = 0.5$. In each figure, the dash line represents the DRC utility function, and four solid lines represent FRT utility functions with various w_1 values. The DRC dash line intersects the FRT solid lines four times, representing four critical demand values. For example, in Fig. 4, the DRC line intersects the FRT ($w_1 = 5$) line at the demand value 49.6 customer/mile²/h which is the critical demand. In this way critical demands are drawn from these figures, and listed in Table 5, where we also list them for other values for α .

The following Table 5 (in which sensitivity over α are also provided and the bold rows are for $\alpha = 0.5$) and Fig. 7 summarize the critical demand results. Fig. 7 shows that with the increase of weight w_1 , the critical values increase. That is, the DRC service is more preferred when planners give larger values of weight for customer walking time. Fig. 7 also shows that the critical values decrease with the increase of length/width ratios. These critical demands are quantitative references for planners to make the decision of operating the feeder service with a FRT or DRC policy, depending on the expected or historic demand in a certain area. Because demand rates may generally fluctuate significantly in a given day, these values also represent “switching points” from one policy to the other.

5.5. Effect of α

The numbers of pick-up and drop-off customers may significantly vary throughout the day, such as in the morning or afternoon peak hours. The sensitivity over α presented in the above Table 5 and Fig. 7. For the FRT service provide an

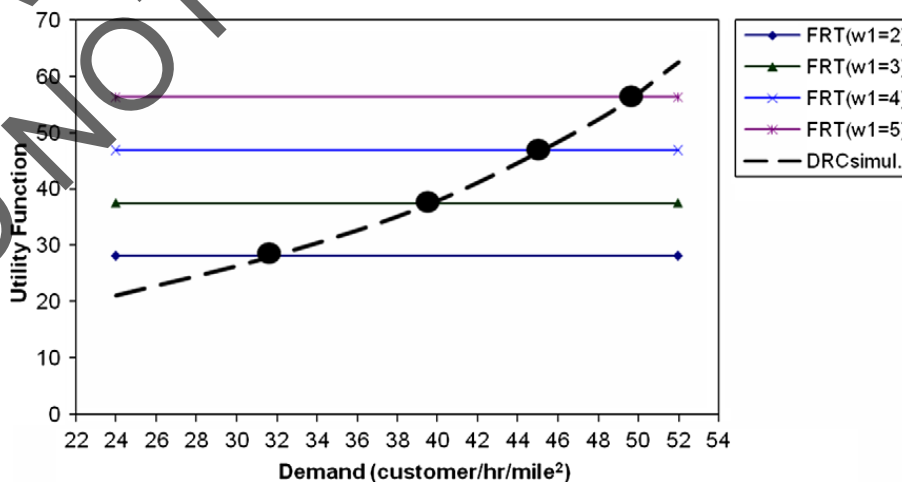


Fig. 4. Comparison of utility function of FRT and DRC ($L = 1, W = 1, \alpha = 0.5$).

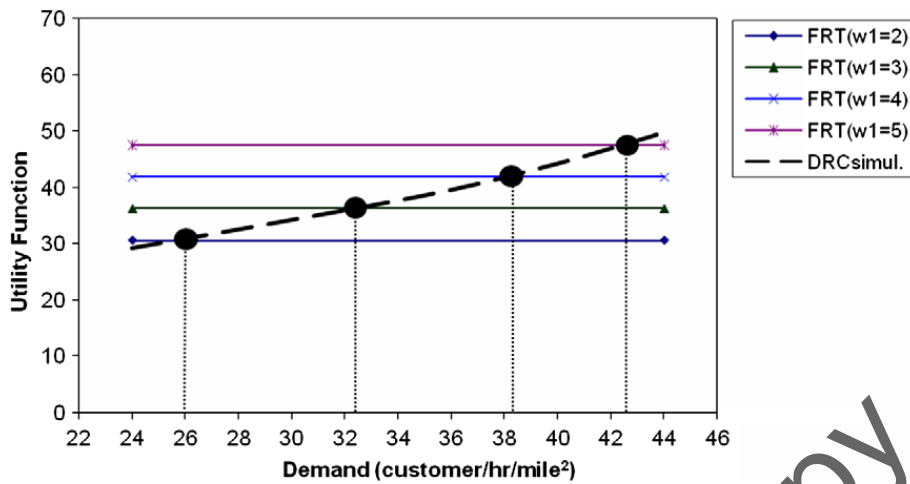


Fig. 5. Comparison of utility function of FRT and DRC ($L = 2, W = 0.5, \alpha = 0.5$).

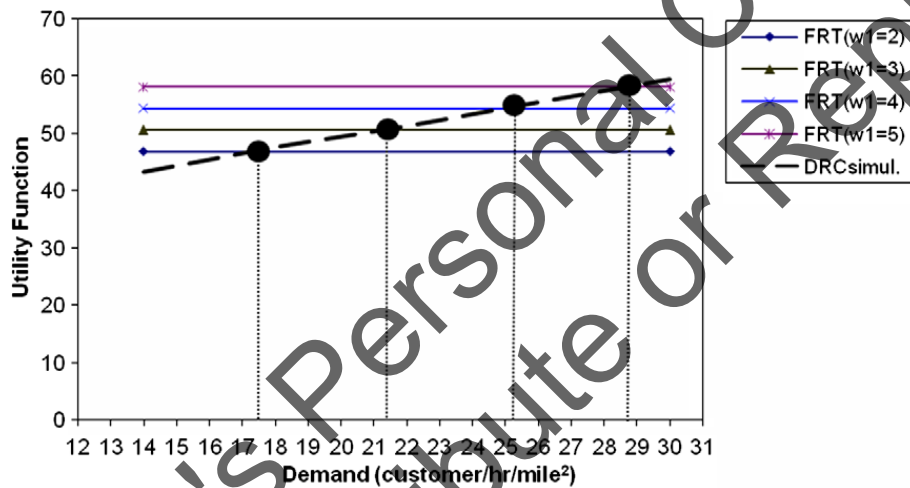


Fig. 6. Comparison of utility function of FRT and DRC ($L = 4, W = 0.25, \alpha = 0.5$).

Table 5
Critical demands (customer/mile²/h) for various w_1, α and L/W ratios.

L/W	α	$w_1 = 2$	$w_1 = 3$	$w_1 = 4$	$w_1 = 5$
1/1	0.1	35.3	42.9	48.7	52.7
	0.3	33.6	41.3	46.8	51
	0.5	31.8	39.7	45.1	49.6
	0.7	30.2	38.2	43.6	48.2
	0.9	28.5	36.6	42.2	46.9
2/0.5	0.1	30.9	37.5	42.4	46.9
	0.3	28.2	34.9	40.1	44.7
	0.5	25.5	32.1	38.1	42.3
	0.7	23.3	30.2	35.6	40.6
	0.9	21.5	27.9	33.8	38.4
4/0.25	0.1	23.9	28	31.8	35.3
	0.3	20.3	24.6	28.3	32
	0.5	17.5	21.1	24.9	28.7
	0.7	15.2	18.4	22	25.6
	0.9	12.9	16	19.1	22.3

interesting insight. We observe from Eqs. (2), (6), and (9) that when α increases the expected values of the customer walking time, ride time and waiting time remain constant. On the other hand, from the simulation results for the DRC service, it is found that the expected value of the customer ride time slightly changes and the expected value of the customer waiting

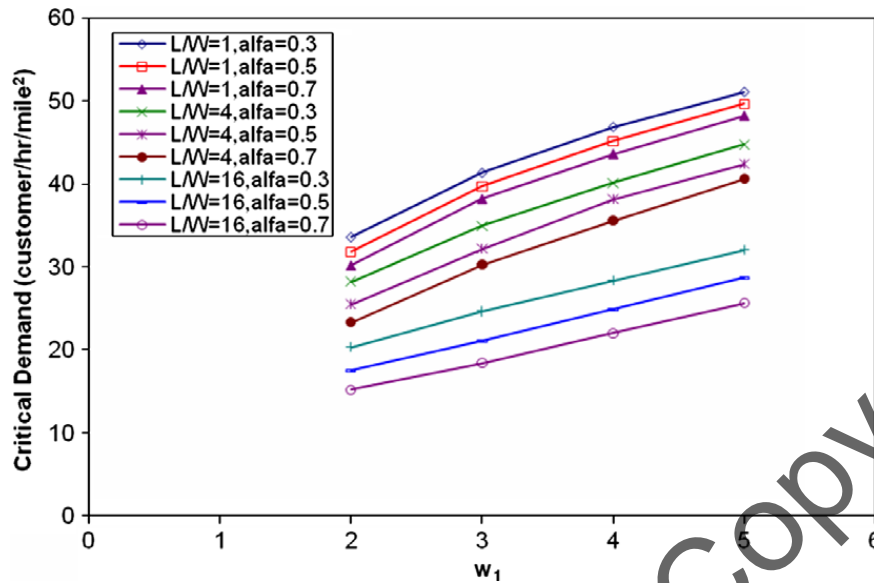


Fig. 7. Critical demands for various L/W ratios and α values.

time increases significantly when α (the portion of pick-up customers) increases, thus worsening the DRC performance. The combination of these effects causes the derived critical customer demands to be larger when the α values are smaller. That is, for a given demand rate, the DRC service performs better when there are more drop-off than pick-up customers (such as in afternoon peak hours of a regular weekday).

6. Conclusions

Feeder transit services are one of the types of flexible transit service most often adopted. Planners often face a difficult decision when having to decide between a demand responsive (DRC) and a fixed-route (FRT) operating policy to adopt in a given area. In this paper, analytical and simulation modeling are developed to create tools to assist planners in this decision making process.

Based on the simulations of one-vehicle operations, we derive critical customer demand densities, representing switching points between the two alternative services, which can be used as a tool for planning and operating decisions. To cover and analyze a wide range of scenarios, we performed sensitivity analysis over various L/W ratios of a rectangular service area, different values for the walking weight, the most susceptible to variation due to external conditions, and over the parameter α , representing the ratio between “pick-up” and “drop-off” customers.

Results show that the DRC service performs better (not surprisingly) with lower demand rates and becomes progressively more preferred when larger values are assigned to the weight for customer walking time, which may correspond to scenarios with unsafe areas or with bad weather conditions. The analysis over α shows that the DRC service performs increasingly better when the portion of drop-off customers increases, such as in afternoon peak hours.

In conclusion, we would like to emphasize that the main purpose of this paper is to provide a general methodology to better conduct the decision making process for feeder transit operations. It is expected that this paper may foster research and applications of more practical oriented scenarios and case studies. Future research may also include an approximate analytical derivation of the MOPs for the DRC case to derive handy analytical tools in addition to the tables provided in this paper.

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Appendix A. Derivation of Eq. (4a)

For pick-up passengers, let X denote the nearest bus station to passengers, $x \in \{1, 2, \dots, N\}$. Let Y denote the ride direction of pick-up passengers at a bus station, $y \in \{1, -1\}$. $Y = 1$ for direction leaving the terminal, and $Y = -1$ for direction approaching the terminal. The Probability Mass Function (pmf) of X is

$$f_x(x) = \begin{cases} \frac{1}{2(N-1)} & \text{for } x = 1 \\ \frac{1}{N-1} & \text{for } x = 2, \dots, N-1 \\ \frac{1}{2(N-1)} & \text{for } x = N-1 \end{cases}$$

The conditional pmf $f(y|x) = P(Y = y|X = x)$ is

$$f(1|x) = \frac{x-1}{N-1}, \quad f(-1|x) = \frac{N-x}{N-1}, \quad \text{for } x = 1, 2, \dots, N-1 \\ f(1|N) = 0, \quad f(-1|N) = 1$$

The ride time of pick-up passengers, $T_{rd}^p = g(X, Y)$. Then

$$g(x, y) = \begin{cases} 0 & \text{for } x = 1 \\ T_c - \frac{(x-1)T_c}{2(N-1)} & \text{for } y = 1; x = 2, \dots, N \\ \frac{(x-1)T_c}{2(N-1)} & \text{for } y = -1; x = 2, \dots, N \end{cases}$$

Therefore,

$$E(T_{rd}^p) = \sum_{x,y} g(x, y)f(x, y) = \sum_{x,y} g(x, y)f(y|x)f_x(x) = \left[\frac{5}{12} - \frac{1}{6(N-1)^2} \right] T_c$$

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